

Math 1231 Fall 2020
Single-Variable Calculus I Mastery Quiz 6
Due midnight on Thursday, October 15

This week's mastery quiz has nine topics. **Do not answer all nine.** You may answer the two first questions on the newest topics, numbered 11 and 10, and *one* additional topic of the previous three. You may pick one topic you have not yet demonstrated mastery on and answer the question on that topic. (If you are retrying a topic, please complete the entire page.)

Don't worry if you make a minor error, but try to demonstrate your mastery of the underlying material. You shouldn't spend more than 10-20 minutes on this quiz. Feel free to consult your notes, but please don't talk about the actual quiz questions with other students in the course.

Remember that you are trying to demonstrate that you understand the concepts involved. For all these problems, justify your answers and explain how you reached them. Do not just write "yes" or "no" or give a single number.

Please upload your work as *one PDF file*. You can produce the file on your computer/tablet/whatever, or you can handwrite it and then scan it. If you have a smartphone, there are many apps that can help you produce a clean single pdf; I personally have used GeniusScan but there are many options.

Topics:

11. Implicit Differentiation
10. Rates of Change
9. Linear Approximations and Tangent Lines
8. Trigonometry and the Chain Rule
7. Basics of Computing Derivatives
6. Definition of a Derivative
5. Infinite Limits
3. Computing Limits
2. Formal limits

11. Implicit Differentiation

- (a) Find an equation for the tangent line to the curve $x^3y + 3xy^2 = 40$ at the point $(2, 2)$.

- (b) Find a formula for y' in terms of x and y if $\sqrt{x+y} = x^3y^2$.

10. Rates of Change

(a) Let $F(x) = 1/x + 1$ be the amount of pressure exerted on a beam in pounds per square inch at a point x inches to the right of its left end.

(i) What does the derivative $F'(x)$ represent, and what are its units?

(ii) Compute $F'(5)$. What does this tell you?

(b) Suppose the height of a particle in centimeters is given as a function of time in seconds $p(t) = t^3 - 3t$.

(i) When is the velocity zero?

(ii) When is the acceleration zero?

9. Linear Approximation and Tangent Lines

(a) Give a formula for a linear approximation of $f(x) = \frac{x}{x-3}$ near the point $a = 4$.

(b) Use your answer in part (a) to estimate $f(3.9)$.

(c) Write the equation for the tangent line to $g(x) = \sin(x^2 - 3x)$ at the point $a = 0$.

8. **Trigonometry and the Chain Rule** Compute:

(a) $\frac{d}{dx} \sec(\tan(\cos((x+1)^2)))$

(b) $\frac{d}{dx} \frac{\sqrt{x^3+x} + \sqrt[3]{x}}{\csc(x)+1}$

7. **Basics of Computing Derivatives** Compute the following derivatives while explicitly naming every derivative rule you use.

(a) $\frac{d}{dx}(x^2 + 1)(x - 3)$

(b) $\frac{d}{dx} \frac{x^2 + 3x}{x^3 - 1} =$

6. Definition of a Derivative

Compute the following derivatives, *directly from the formal definition of derivative*.

(a) If $f(x) = \frac{x+1}{x-1}$, find $f'(2)$.

(b) If $g(x) = \sqrt{x-5}$, find $g'(x)$.

5. **Infinite Limits** Compute:

$$(a) \lim_{x \rightarrow -\infty} \frac{\sqrt{x^2 + x + 1}}{x + 3} =$$

$$(b) \lim_{x \rightarrow 3} \frac{x + 3}{x^2 - 9} =$$

$$(c) \lim_{x \rightarrow 2} \frac{x - 1}{(x - 2)^2} =$$

3. **Computing Limits** Compute:

$$(a) \lim_{x \rightarrow 1} \frac{\sqrt{x+3} - 2}{x-1} =$$

$$(b) \lim_{x \rightarrow 3} \frac{x^2 - 5x + 6}{x^2 - x - 6} =$$

$$(c) \lim_{x \rightarrow 0} \frac{1}{x} - \frac{1}{x^2 + x} =$$

2. Formal Limits

(a) Write a formal ϵ - δ proof that $\lim_{x \rightarrow 3} 3x - 1 = 8$.

(b) Explicitly naming each limit law you use, compute

$$\lim_{x \rightarrow -1} \frac{\sqrt{x^2 + 3}}{2x + 3} =$$