

Math 1231 Fall 2020  
Single-Variable Calculus I Mastery Quiz 9  
Due midnight on Thursday, November 5

This week's mastery quiz has eight topics. **Do not answer all ten.** You may answer the two first questions on the newest topics, numbered 15 and 14, and *one* additional topic. You may pick one topic you have not yet demonstrated mastery on and answer the question on that topic. (If you are retrying a topic, please complete the entire page.)

Don't worry if you make a minor error, but try to demonstrate your mastery of the underlying material. You shouldn't spend more than 10-20 minutes on this quiz. Feel free to consult your notes, but please don't talk about the actual quiz questions with other students in the course.

Remember that you are trying to demonstrate that you understand the concepts involved. For all these problems, justify your answers and explain how you reached them. Do not just write "yes" or "no" or give a single number.

Please upload your work as *one PDF file*. You can produce the file on your computer/tablet/whatever, or you can handwrite it and then scan it. If you have a smartphone, there are many apps that can help you produce a clean single pdf; I personally have used GeniusScan but there are many options.

Topics:

15. Curve Sketching
14. First and Second Derivative Tests
13. Global Maxima and Critical Points
12. Related Rates
11. Implicit Differentiation
10. Rates of Change
4. Trigonometric Limits
1. Informal limits and continuity

### 15. Curve Sketching

Let  $f(x) = \frac{(x-2)^2}{x-1}$ . We can compute that

$$f'(x) = \frac{x(x-2)}{(x-1)^2}$$
$$f''(x) = \frac{2}{(x-1)^3}.$$

Sketch a graph of  $f$ . Your answer should discuss the domain, roots, asymptotes, limits at infinity, critical points and values, intervals of increase and decrease, and concavity and points of inflection.

#### 14. First and Second Derivative Tests

(a) Classify the critical points and relative extrema of  $f(x) = 5 + 8x^3 + x^4$ .

(b) Classify the critical points and relative extrema of  $g(x) = \frac{2x - 1}{x^2 + 2}$ .

### 13. Global Maxima and Critical Points

- (a) Find the absolute extrema of  $g(x) = x^3 - 3x^2 - 9x + 5$  on  $[-2, 4]$ , and justify your claim that these are in fact absolute extrema.

- (b) Find all the critical points of  $g(x) = \frac{x^2 - 3x - 4}{x + 5}$

## 12. Related Rates

A rocket is taking off with a perfectly vertical path, and is being tracked by a radar station on the ground four miles from the launch pad. How fast is the rocket rising when it is three miles high and its distance from the radar station is increasing at a rate of 3000 miles per hour.

## 11. Implicit Differentiation

(a) Write a tangent line to the curve  $x^2y = x + 2y$  at the point  $(2, 1)$ .

(b) Find a formula for  $y'$  in terms of  $x$  and  $y$  if  $xy = x^2 \sin(y)$ .

## 10. Rates of Change

(a) The force a magnet exerts on a piece of iron depends on the distance between the magnet and the metal. Let  $F(d) = \frac{2}{d^2}$  give the force exerted by the magnet in Newtons, where  $d$  is the distance between them in meters.

(i) What does the derivative  $F'(d)$  represent, and what are its units?

(ii) Calculate  $F'(2)$ . What does this tell you physically?

(b) Suppose the distance between two particles in centimeters is given as a function of time in seconds by the formula  $d(t) = t + \frac{1}{t}$ .

(i) When is the velocity zero?

(ii) When is the acceleration zero?

## 9. Linear Approximation and Tangent Lines

(a) Use a linear approximation to estimate  $\sqrt[3]{8.3}$ .

(b) Find an equation of the line tangent to  $y = x \tan x$  at the point  $x = \pi/4$ .

#### 4. Trigonometric Limits

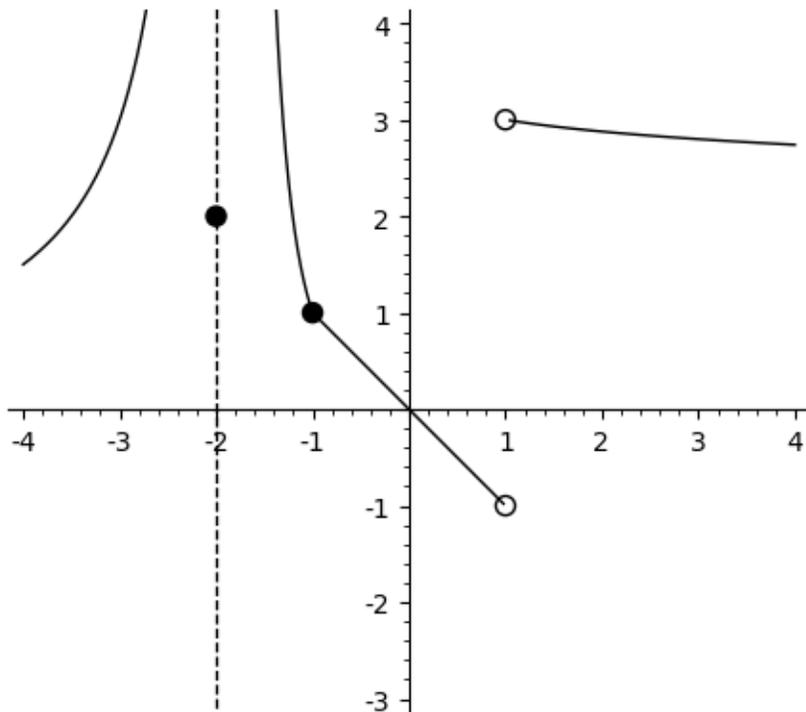
(a) Show that  $\lim_{x \rightarrow 3} |x - 3| \cos\left(\frac{x}{x - 3}\right) = 0$ .

(b) Compute  $\lim_{x \rightarrow 0} \frac{\sin(x) \sin(2x) \sin(3x)}{x^3}$ .

## 1. Informal Continuity and Limits

(a) Give a (zeroth-order) approximate value for  $\cos(3)$ , and explain how you got it.

Here is the graph of a function  $f$ :



For each of the following questions, if your answer is “does not exist”, explain in a few words why it does not exist. If your answer is just a number, you don’t need to explain.

- (b) What is the domain of  $f$ ?
- (c) Where (if anywhere) is  $f$  discontinuous?
- (d) What is  $\lim_{x \rightarrow -1} f(x)$ ?
- (e) What is  $f(-1)$ ?
- (f) What is  $\lim_{x \rightarrow 1} f(x)$ ?
- (g) What is  $f(1)$ ?