Vectors

1) list of numbers (Algebraic)

$$\begin{bmatrix} 1 \\ 3 \\ 2 \\ 4 \end{bmatrix}$$

Computer science
Statistics
Machine Learning

2) arrow (geometry)

physics

3) properties (formal)

$\vec{u}+\vec{v} = \vec{v}+\vec{u}$

$r(\vec{u}+\vec{v}) = r\vec{u}+r\vec{v}$

$\vec{0}+\vec{u} = \vec{u}$

math
Quantum mechanics
Statistics

$3x+y=4$

$2x+5y=7$

$x=1, y=1$ solution is $[1]$

set of solutions is $\{[1]\}$

$x+3y=-5$

$z=3$

$x+3y+4z=7$

$3x+9y+7z=6$

$$\begin{bmatrix} 1 & 3 & 4 & 7 \\ 3 & 9 & 7 & 6 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & 3 & 4 & 7 \\ 0 & 0 & -5 & -15 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & 3 & 0 & -5 \\ 0 & 0 & 1 & 3 \end{bmatrix}$$

$$\begin{bmatrix} -5 \\ 0 \\ 3 \end{bmatrix}, \begin{bmatrix} -8 \\ 1 \\ 3 \end{bmatrix}, \begin{bmatrix} -11 \\ 2 \\ 3 \end{bmatrix}, \dots$$

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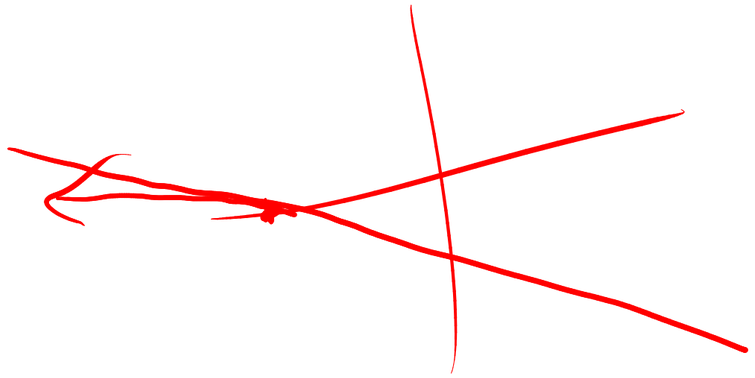
$$\left\{ \begin{bmatrix} -5-3y \\ y \\ 3 \end{bmatrix} \middle| y \in \mathbb{R} \right\} = \left\{ \begin{bmatrix} x \\ \frac{-5-x}{3} \\ 3 \end{bmatrix} \middle| x \in \mathbb{R} \right\}$$

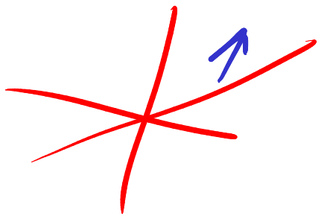
$$\left\{ \begin{bmatrix} -5-3y \\ y \\ 3 \end{bmatrix} \middle| \, y \in \mathbb{R} \right\} = \begin{bmatrix} -5 \\ 0 \\ 3 \end{bmatrix} + y \begin{bmatrix} -3 \\ 1 \\ 0 \end{bmatrix}$$

$$= \begin{bmatrix} -5 \\ 0 \\ 3 \end{bmatrix} + \begin{bmatrix} -3y \\ y \\ 0 \end{bmatrix}$$



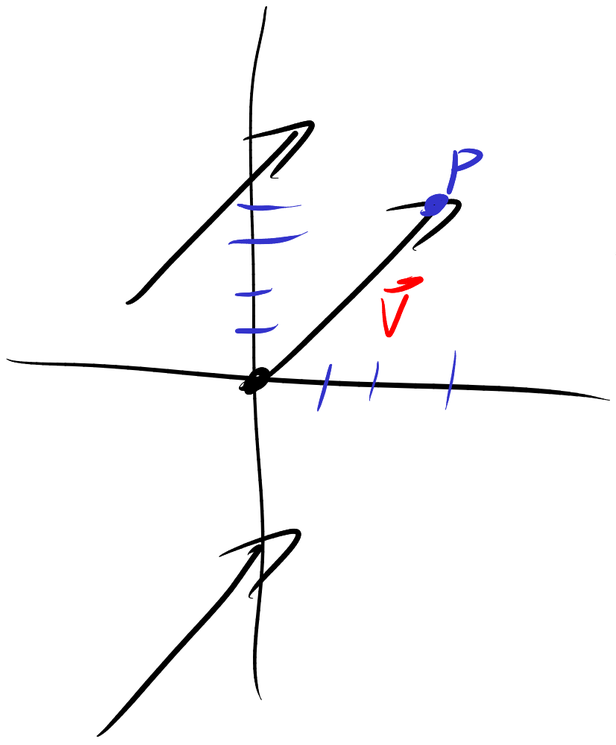
$$\left\{ \begin{bmatrix} x \\ y \\ x+y \end{bmatrix} \;\middle|\; x,y \in \mathbb{R} \right\} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} + x \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix} + y \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix}$$

$\subseteq \mathbb{R}^3$



$$\left\{ \begin{bmatrix} x \\ y \\ z \end{bmatrix} \;\middle|\; x,y,z \in \mathbb{R} \right\} \subseteq \mathbb{R}^3$$

$$\left\{ \begin{bmatrix} x \\ y \\ z \\ x+y \\ 1+y+z \end{bmatrix} \;\middle|\; x,y,z \in \mathbb{R} \right\} \subseteq \mathbb{R}^5$$



$P = (3, 4)$

$\vec{v} = \begin{bmatrix} 3 \\ 4 \end{bmatrix}$

$\begin{bmatrix} 1 \\ 2 \\ 3 \\ 4 \\ 5 \\ 6 \end{bmatrix}$

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what is the angle btwn ↷

what is angle btwn $\begin{bmatrix} 1 \\ 3 \\ 4 \\ 7 \\ 2 \end{bmatrix}$ $\begin{bmatrix} 2 \\ 1 \\ 5 \\ 1 \\ 6 \end{bmatrix}$

what is angle btwn $\sin(x)$ and $\cos(x)$

$90°$

Linear Combinations

a linear combination of

$$\begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix}$$

i.e. $a\begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix} + b\begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix}$.

$$\begin{bmatrix} 1 \\ 3 \\ 4 \end{bmatrix} \text{ in span } \begin{bmatrix} e \\ -\pi \\ e-\pi \end{bmatrix}$$

Span

if $S = \{\vec{v}_1, \vec{v}_2, \vec{v}_3, \ldots, \vec{v}_n\}$

the span of S is

set of all LCs.

given $S = \{\vec{v}_1, \ldots, \vec{v}_n\}$

$\vec{b}$ a vector,

is $\vec{b} \in \text{span } S$?

is $\begin{bmatrix} 1 \\ -7 \\ -5 \end{bmatrix} \in \mathbb{R}^3$ in span of $\left\{ \begin{bmatrix} 6 \\ -1 \\ 5 \end{bmatrix}, \begin{bmatrix} -3 \\ 4 \\ 0 \end{bmatrix} \right\}$ ($\mathbb{R}^3$, $\in \mathbb{R}^3$)

$$\begin{bmatrix} 6a - 3b \\ -a + 4b \\ 5a + 0b \end{bmatrix} = a \begin{bmatrix} 6 \\ -1 \\ 5 \end{bmatrix} + b \begin{bmatrix} -3 \\ 4 \\ 0 \end{bmatrix} = \begin{bmatrix} 1 \\ -7 \\ -5 \end{bmatrix}$$

$$\left[\begin{array}{cc|c} 6 & -3 & 1 \\ -1 & 4 & -7 \\ 5 & 0 & -5 \end{array}\right]$$

$6a - 3b = 1 \quad -3b = 7$

$-a + 4b = -7$ ☹

$5a + 0b = -5$

$a = -1$

$6a - 3b = 1$

$-a + 4b = -7$

$5a + 0b = -5$

are the same

$$\begin{bmatrix} 6 & -3 \\ -1 & 4 \\ 5 & 0 \end{bmatrix} \begin{bmatrix} a \\ b \end{bmatrix} = \begin{bmatrix} 1 \\ -7 \\ -5 \end{bmatrix}.$$

Matrix equation version

1) is $\vec{b}$ in span S

2) can we get to b w/ vectors in S

3) is there a solution to vector eqn $a_1\vec{v}_1 + a_2\vec{v}_2 = \vec{b}$

4) is there a soln to linear system

5) is there a soln to matrix eqn $A\vec{x} = \vec{b}$

6) if I row reduce the matrix, every non-zero row has a non-zero entry outside the right column.

$\left[\begin{array}{ccc|c} & & & \\ & & & \\ & & & \end{array}\right]$ | is $\begin{bmatrix} 1 \\ 0 \end{bmatrix}$ in span $\left\{ \begin{bmatrix} 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 2 \\ 0 \end{bmatrix}, \begin{bmatrix} 3 \\ 0 \end{bmatrix}, \begin{bmatrix} 4 \\ 0 \end{bmatrix} \right\}$