Math 2184 Final

Instructor: Jay Daigle

1. This test is due Tuesday, December 15 at 2:40 PM. Logistically, this will work just like the mastery quizzes: download it, write up your answers, and upload them to Blackboard for us to grade.
2. You will have two hours for this test. Please write down your start and end times on the test and include that in your upload. You may not spend more than two hours on the test unless you have a specific accommodation.
3. You are not allowed to consult books or notes during the test, but you may use a one-page cheat sheet you have made for yourself ahead of time. Please upload your sheet along with your test.
4. If you have questions, I will be online and responsive on the course Blackboard from 12:40 - 2:40. If you want to take the test at a time you know I'll be able to answer any questions quickly, I encourage you to use that time slot. I will also be on discord and email as frequently as I reasonably can.
5. You may use a calculator, but don't use anything that has the ability to do row-reductions for you. Using a calculator for basic arithmetic is fine.
6. The maximum score for this test is 80 points.

Name:

Time Started:

Time Completed:

Problem 1. (a) Let $A = \begin{bmatrix} 3 & 6 & 4 \\ 3 & -1 & -3 \\ 0 & 7 & 7 \end{bmatrix}$. Find the characteristic polynomial, the eigenvalues, and a basis for each eigenspace.

(b) Find the determinant and trace of $B = \begin{bmatrix} 2 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 \\ 1 & 6 & 2 & 0 \\ 1 & 1 & -2 & 3 \end{bmatrix}$ and $C = \begin{bmatrix} 1 & 3 & 2 \\ 4 & 1 & -2 \\ 2 & 1 & 3 \end{bmatrix}$.

Problem 2. (a) Let

$$E = \left\{ \begin{bmatrix} 2 \\ -5 \\ 1 \end{bmatrix}, \begin{bmatrix} 3 \\ 1 \\ 2 \end{bmatrix}, \begin{bmatrix} 4 \\ -7 \\ 3 \end{bmatrix} \right\} \quad \text{and} \quad F = \left\{ \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix} \right\}.$$

Find the change of basis matrix from E to F.

(b) Thhe matrix $A = \begin{bmatrix} 1 & 2 & -1 \\ 2 & 4 & -2 \\ 3 & 6 & -3 \end{bmatrix}$ has eigenvalues 2 and 0. Diagonalize it, and compute A^5.

Problem 3.

(a) Let $V = \mathcal{P}_2(x)$, with the inner product $\langle f, g \rangle = f(-1)g(-1) + f(1)g(1) + f(3)g(3)$. Compute $\|3 - x\|$ and $\|1 + x^2\|$. Find the projection of 1 onto $3 - x$. Find the projection of 1 onto $1 + x^2$.

(b) Let $U = \text{span}\{(2, 1, 1, 0), (1, -2, 0, 1)\}$. Find an orthonormal basis for $U^\perp$, and then find the orthogonal decomposition of $(3, -2, 2, -1)$ with respect U.

Problem 4. Let $A = \begin{bmatrix} 1 & 2 \\ 1 & 2 \\ 2 & 3 \\ 2 & 5 \end{bmatrix}$ and $\mathbf{b} = \begin{bmatrix} 2 \\ 4 \\ -2 \\ 4 \end{bmatrix}$.

(a) Find a least-squares solution $\hat{\mathbf{x}}$ to the equation $A\mathbf{x} = \mathbf{b}$.

(b) Compute $\|\mathbf{b} - A\hat{\mathbf{x}}\|$. What does this tell you about your solution from part (a)?