Problem 1. (a) Let $A = \begin{bmatrix} 1 & -1 & -1 \\ 0 & 2 & 0 \\ -1 & -1 & 1 \end{bmatrix}$. Find the characteristic polynomial, the eigenvalues, and a basis for each eigenspace.

(b) Find the determinant and trace of $B = \begin{bmatrix} 1 & -1 & 0 & 3 \\ 2 & 5 & 2 & 6 \\ 0 & 1 & 0 & 0 \\ 1 & 4 & 2 & 1 \end{bmatrix}$ and $C = \begin{bmatrix} 1 & 2 & 3 \\ 0 & 4 & 1 \\ 1 & 6 & 4 \end{bmatrix}$.

Problem 2. (a) Let

$$E = \left\{ \begin{bmatrix} 3 \\ 0 \\ 1 \end{bmatrix}, \begin{bmatrix} -1 \\ 3 \\ 2 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix} \right\} \qquad \text{and} \qquad F = \left\{ \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} \right\}.$$

Find the change of basis matrix from E to F.

(b) Diagonalize the matrix $A = \begin{bmatrix} 2 & 0 & 1 \\ 1 & 1 & 1 \\ 1 & 0 & 2 \end{bmatrix}$ and compute A^5.

Problem 3.

(a) Let $V = \mathcal{C}([1,3],\mathbb{R})$, with the inner product $\langle f, g\rangle = \int_1^3 f(t)g(t)\,dt$. Find $\|1\|$ and $\|x\|$. Find the projection of $1 + x$ onto 1 and onto x.

(b) Let $U = \text{span}\{(1,1,1,0),(1,0,-1,1)\}$. Find an orthonormal basis for $U^{\perp}$, and then find the orthogonal decomposition of $(2,-1,5,6)$ with respect U.

Problem 4. Let $A = \begin{bmatrix} 1 & -2 \\ -1 & 2 \\ 0 & 3 \\ 2 & 5 \end{bmatrix}$ and $\mathbf{b} = \begin{bmatrix} 3 \\ 1 \\ -4 \\ 2 \end{bmatrix}$.

(a) Find a least-squares solution $\hat{\mathbf{x}}$ to the equation $A\mathbf{x} = \mathbf{b}$.

(b) Compute $(\mathbf{b} - A\hat{\mathbf{x}}) \cdot A\hat{\mathbf{x}}$. What does this tell you about your solution from part (a)?