Problem 1.

(a) (10 points) Find the set of solutions to the following system of linear equations:

$$\begin{aligned} 3x + 7y + 5z &= 34 \\ 2x + 4y + 2z &= 20 \\ -x + 3z &= -2 \end{aligned}$$

(b) (10 points) Prove that the following vectors in $\mathcal{P}_2(x)$ are linearly independent, *explicitly using the formal definition of linear independence*:

$$f(x) = 1 + x, g(x) = 3 - x + x^2, h(x) = 2 + 3x^2$$

Problem 2.

(a) (10 points) Find the inverse of the matrix $\begin{bmatrix} 2 & -2 & 3 \\ 2 & 2 & 2 \\ 1 & 2 & 1 \end{bmatrix}$.

(b) (5 points) If $B^{-1} = \begin{bmatrix} 2 & 3 & 1 \\ 6 & 2 & -2 \\ 4 & 1 & 1 \end{bmatrix}$ find the set of solutions to $B\mathbf{x} = \begin{bmatrix} 4 \\ 3 \\ 2 \end{bmatrix}$.

(c) (5 points) Compute

$$\begin{bmatrix} 3 & 2 & 1 \\ 2 & 5 & 3 \end{bmatrix} \begin{bmatrix} 5 & -1 \\ 4 & 2 \\ -3 & 3 \end{bmatrix}$$

Problem 3 (10 points each).

(a) Prove that $T = \{a_0 + a_1 x + a_2 x^2 : a_0 = a_1\}$ is a subspace of $\mathcal{P}_2(x)$.

(b) Is $(3, 2, 5)$ in the span of the set $S = \{(1, 1, 1), (1, 2, 3), (3, 5, 7)\}$?

Problem 4.

Let $T : \mathcal{P}_2(x) \to \mathbb{R}^3$ be given by $T(f) = \begin{bmatrix} f(0) \\ f(2) - f(0) \\ 2f(-2) \end{bmatrix}$.

(a) Prove T is a linear transformation.

(b) Find a matrix for T with respect to the standard bases $\{1, x, x^2\}$ and $\{(1,0,0), (0,1,0), (0,0,1)\}$ for each space.

(c) Find a basis for $\ker(T)$.

(d) Find a basis for the image $T(\mathcal{P}_2(x))$.

(e) If T is invertible, find a (non-matrix!) formula for the inverse.