Math 2184 §10 Fall 2020
Linear Algebra I Mastery Quiz 11
Due Midnight on Friday, November 20

This week's mastery quiz has seven topics. **Do not answer all questions.** Please answer the problems on the new topics, labeled 19 and 18. You may answer *one* of the other topics if you did not get a "mastery" grade on them already. If you are retrying a topic you should complete the entire page.

Don't worry if you make a minor error, but try to demonstrate your mastery of the underlying material. You shouldn't spend more than 30 minutes on this quiz. Feel free to consult your notes, but please don't talk about the actual quiz questions with other students in the course.

Remember that you are trying to demonstrate that you understand the concepts involved. For all these problems, justify your answers and explain how you reached them. Do not just write "yes" or "no" or give a single number.

Please upload your work as *one PDF file*. You can produce the file on your computer/tablet/whatever, or you can handwrite it and then scan it. If you have a smartphone, there are many apps that can help you produce a clean single pdf; I personally have used GeniusScan but there are many options.

Topics are:

19. Dot Product and Projection
18. Diagonalization
17. Similarity and Trace
16. Change of Basis
15. Complex and Generalized Eigenvectors
7. Subspaces
4. Linear Transformations

19. **Dot Product and Projection** Let $\mathbf{u} = (1, 4, 2)$ and $\mathbf{v} = (-3, 7, 1)$.

(a) Compute $\|\mathbf{u}\|$ and $\|\mathbf{v}\|$.

(b) Compute $\operatorname{Proj}_{\mathbf{v}} \mathbf{u}$ and the projection of $\mathbf{u}$ orthogonal to $\mathbf{v}$.

(c) What is the angle between $\mathbf{u}$ and $\mathbf{v}$?

(d) Find a vector orthogonal to both $\mathbf{u}$ and $\mathbf{v}$.

18. **Diagonalization** Let $A = \begin{bmatrix} -1 & 0 & 1 \\ -2 & 0 & 2 \\ -2 & 1 & 1 \end{bmatrix}$. Diagonalize A, and use this diagonalization to compute A^{100}.

17. **Similarity and Trace**

(a) Let $A = \begin{bmatrix} 8 & 10 \\ -3 & -3 \end{bmatrix}$, $B = \begin{bmatrix} 6 & 3 \\ -4 & -1 \end{bmatrix}$. Show that A is similar to B.

(b) Let $C = \begin{bmatrix} 4 & 3 \\ 2 & 1 \end{bmatrix}$ and $D = \begin{bmatrix} 3 & 2 \\ 1 & -6 \end{bmatrix}$. Show that neither matrix is similar to A.

16. **Change of Basis**

Let

$$E = \left\{ \begin{bmatrix} 2 \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 4 \\ 1 \\ 2 \end{bmatrix}, \begin{bmatrix} 5 \\ 1 \\ 3 \end{bmatrix} \right\}, \qquad F = \left\{ \begin{bmatrix} 1 \\ 3 \\ 0 \end{bmatrix}, \begin{bmatrix} 2 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 3 \\ 3 \\ 1 \end{bmatrix} \right\}.$$

Find the transition matrix from E to F.

15. **Complex and Generalized Eigenvectors**

(a) Let $A = \begin{bmatrix} 2 & -2 \\ 1 & 4 \end{bmatrix}$. Find all (complex!) eigenvalues of A and find a (possibly complex) eigenvector for each eigenvalue.

(b) Let $B = \begin{bmatrix} 4 & 4 & 4 \\ -11 & -8 & -7 \\ 10 & 8 & 7 \end{bmatrix}$. Find an eigenvector of eigenvalue 2, and then find a generalized eigenvector of eigenvalue 2.

7. **Subspaces** For each of the following sets, determine whether it is a subspace of $\mathbb{R}^3$, and justify your answer using the definition of subspace.

   (a) $U = \{(x, y, z) : 3x + 5y + z = 0\}$

   (b) $V = \{(x, y, z) : 3x - 2z + 4 = y\}$

   (c) $W = \{(x, y, z) : xyz = xy + xz\}$.

4. **Linear Transformations**

(a) Prove directly from the definition that the following function is linear:

$$L\left(\begin{bmatrix} x \\ y \\ z \end{bmatrix}\right) = \begin{bmatrix} x - y \\ 3x + z \\ y + z \end{bmatrix}$$

(b) Write a matrix for the linear function.

(c) Is this function one-to-one, onto, both, or neither? Why?