

Math 2184 §10 Fall 2020
Linear Algebra I Mastery Quiz 1
Due Noon on Tuesday, September 8

This week's mastery quiz has two topics. Please do your best on each topic. Don't worry if you make a minor error, but try to demonstrate your mastery of the underlying material. You shouldn't spend more than 20-30 minutes on this quiz.

Feel free to consult your notes, but please don't talk about the actual quiz questions with other students in the course.

Remember that you are trying to demonstrate that you understand the concepts involved. For all these problems, justify your answers and explain how you reached them. Do not just write "yes" or "no" or give a single number.

Please upload your work as *one PDF file*. You can produce the file on your computer/tablet/whatever, or you can handwrite it and then scan it. If you have a smartphone, there are many apps that can help you produce a clean single pdf; I personally have used GeniusScan but there are many options.

2. Vector Equations and Spans

- (a) Is the vector $\mathbf{a} = \begin{bmatrix} 5 \\ 5 \\ 5 \end{bmatrix}$ in the span of the vectors $\mathbf{u}_1 = \begin{bmatrix} 1 \\ 2 \\ 4 \end{bmatrix}$, $\mathbf{u}_2 = \begin{bmatrix} 3 \\ 1 \\ -1 \end{bmatrix}$?

Solution: No. We set up the equation

$$x \begin{bmatrix} 1 \\ 2 \\ 4 \end{bmatrix} + y \begin{bmatrix} 3 \\ 1 \\ -1 \end{bmatrix} = \begin{bmatrix} 5 \\ 5 \\ 5 \end{bmatrix}$$

which gives the matrix

$$\begin{aligned} \begin{bmatrix} 1 & 3 & 5 \\ 2 & 1 & 5 \\ 4 & -1 & 5 \end{bmatrix} &\rightarrow \begin{bmatrix} 1 & 3 & 1 \\ 0 & -5 & -5 \\ 0 & -13 & -15 \end{bmatrix} \\ &\rightarrow \begin{bmatrix} 1 & 3 & 1 \\ 0 & 1 & 1 \\ 0 & 13 & 15 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & 0 & -2 \\ 0 & 1 & 1 \\ 0 & 0 & 2 \end{bmatrix} \end{aligned}$$

which is inconsistent.

- (b) Write the vector $\mathbf{b} = \begin{bmatrix} 4 \\ 3 \\ -7 \end{bmatrix}$ as a linear combination of the vectors

$$\mathbf{v}_1 = \begin{bmatrix} 5 \\ 3 \\ 1 \end{bmatrix}, \mathbf{v}_2 = \begin{bmatrix} 2 \\ 1 \\ 3 \end{bmatrix}.$$

Solution: $\begin{bmatrix} 4 \\ 3 \\ -7 \end{bmatrix} = 2 \begin{bmatrix} 5 \\ 3 \\ 1 \end{bmatrix} - 3 \begin{bmatrix} 2 \\ 1 \\ 3 \end{bmatrix}.$

1. Systems of Linear Equations

Use row reduction to find all solutions to each system of equations.

(a)

$$\begin{aligned}4x + 2y + z &= 4 \\x + 3y + z &= 3 \\-2x + 4y + 2z &= 3\end{aligned}$$

Solution: $x = 1/2, y = 1/2, z = 1$

(b)

$$\begin{aligned}2x + 4y + z &= -4 \\3x + 5y + 2z &= -3 \\x + y + z &= 1\end{aligned}$$

Solution: $x = 4 - 3z/2, y = -3 + z/2$, or $\{(4 - 3z/2, -3 + z/2, z)\}$ or

$$\left\{ \begin{bmatrix} 4 \\ -3 \\ 0 \end{bmatrix} + z \begin{bmatrix} -3/2 \\ 1/2 \\ 1 \end{bmatrix} \right\}$$