Math 2184 §10 Fall 2020
Linear Algebra I Mastery Quiz 2
Due Noon on Tuesday, September 15

This week's mastery quiz has four topics. Please answer the problems on the new topics, labeled 4 and 3. You may answer *one* of the other two topics if you did not get a "mastery" grade on it last week.

Don't worry if you make a minor error, but try to demonstrate your mastery of the underlying material. You shouldn't spend more than 30 minutes on this quiz. Feel free to consult your notes, but please don't talk about the actual quiz questions with other students in the course.

Remember that you are trying to demonstrate that you understand the concepts involved. For all these problems, justify your answers and explain how you reached them. Do not just write "yes" or "no" or give a single number.

Please upload your work as *one PDF file*. You can produce the file on your computer/tablet/whatever, or you can handwrite it and then scan it. If you have a smartphone, there are many apps that can help you produce a clean single pdf; I personally have used GeniusScan but there are many options.

Topics are:

4. Linear Transformations
3. Linear Independence
2. Vector Equations and Spanning
1. Systems of Linear Equations

4. **Linear Transformations** Consider the following functions:

$$f\left(\begin{bmatrix} x \\ y \\ z \end{bmatrix}\right) = \begin{bmatrix} 3x + y \\ y - z \\ x + z \end{bmatrix} \qquad g\left(\begin{bmatrix} x \\ y \\ z \end{bmatrix}\right) = \begin{bmatrix} x + 1 \\ y - z \\ z^2 \end{bmatrix}$$

(a) One of these two functions is linear. Which one is linear, and why?

(b) Write a matrix for the linear function.

(c) Is this function one-to-one, onto, both, or neither? Why?

3. **Linear Independence**

(a) *Using the formal definition of linear independence*, show that the set

$$\left\{ \mathbf{u} = \begin{bmatrix} 5 \\ 1 \\ -1 \end{bmatrix}, \mathbf{v} = \begin{bmatrix} 1 \\ 2 \\ -3 \end{bmatrix}, \mathbf{w} = \begin{bmatrix} 2 \\ 1 \\ 4 \end{bmatrix} \right\}$$

is linearly independent.

(b) Find a linear dependence relationship among the following vectors. That is, write one as a linear combination of the others.

$$\left\{ \mathbf{x} = \begin{bmatrix} 2 \\ 4 \\ 1 \end{bmatrix}, \mathbf{y} = \begin{bmatrix} 1 \\ -3 \\ 2 \end{bmatrix}, \mathbf{z} = \begin{bmatrix} 6 \\ 2 \\ 6 \end{bmatrix} \right\}$$

2. **Vector Equations and Spans**

(a) Is the vector $\mathbf{v} = \begin{bmatrix} 3 \\ 0 \\ 3 \end{bmatrix}$ in the span of the vectors $\mathbf{u}_1 = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}, \mathbf{u}_2 = \begin{bmatrix} -1 \\ 1 \\ 0 \end{bmatrix}$?

(b) Write the vector $\mathbf{b} = \begin{bmatrix} 4 \\ 2 \\ 3 \end{bmatrix}$ as a linear combination of the vectors

$\mathbf{v}_1 = \begin{bmatrix} -2 \\ 4 \\ 1 \end{bmatrix}, \mathbf{v}_2 = \begin{bmatrix} 1 \\ 3 \\ 2 \end{bmatrix}$.

1. **Systems of Linear Equations**

   Use row reduction to find all solutions to each system of equations.

   (a)

$$\begin{aligned} 3x + y + 2z &= 1 \\ x + 3y + 2z &= -1 \\ 4x + 4y + 4z &= 0 \end{aligned}$$

   (b)

$$\begin{aligned} 2x + 1y - z &= 2 \\ x + 5y + 2z &= 1 \\ -3x - 2y + z &= 3 \end{aligned}$$