Math 2184 §10 Fall 2020
Linear Algebra I Mastery Quiz 3
Due Midnight on Thursday, September 24

This week's mastery quiz has six topics. **Do not answer all questions.** Please answer the problems on the new topics, labeled 6 and 5. You may answer *one* of the other four topics if you did not get a "mastery" grade on it already.

Don't worry if you make a minor error, but try to demonstrate your mastery of the underlying material. You shouldn't spend more than 30 minutes on this quiz. Feel free to consult your notes, but please don't talk about the actual quiz questions with other students in the course.

Remember that you are trying to demonstrate that you understand the concepts involved. For all these problems, justify your answers and explain how you reached them. Do not just write "yes" or "no" or give a single number.

Please upload your work as *one PDF file.* You can produce the file on your computer/tablet/whatever, or you can handwrite it and then scan it. If you have a smartphone, there are many apps that can help you produce a clean single pdf; I personally have used GeniusScan but there are many options.

Topics are:

6. Matrix Inverses
5. Matrix Multiplication
4. Linear Transformations
3. Linear Independence
2. Vector Equations and Spanning
1. Systems of Linear Equations

6. **Matrix Inverses**

(a) Find the inverse of $A = \begin{bmatrix} 3 & 1 & 2 \\ 2 & 1 & 1 \\ 1 & 0 & 2 \end{bmatrix}$.

(b) Which of the following matrices are invertible? Justify your answers.

$$\begin{bmatrix} 3 & 1 & 2 \\ 5 & 1 & 4 \end{bmatrix} \qquad \begin{bmatrix} 2 & 1 \\ 4 & 2 \end{bmatrix} \qquad \begin{bmatrix} 2 & 1 \\ 4 & 3 \end{bmatrix} \qquad \begin{bmatrix} 1 & 5 & 3 \\ 2 & 1 & 4 \\ 2 & 2 & 3 \end{bmatrix}$$

5. **Matrix Multiplication**

Compute, or write that the computation is undefined:

(a) $\begin{bmatrix} 1 & 3 \\ 2 & 4 \end{bmatrix} \begin{bmatrix} 5 & -2 & 1 \\ 1 & 4 & 4 \end{bmatrix} =$

(b) $\begin{bmatrix} 5 & -2 & 1 \\ 1 & 4 & 4 \end{bmatrix} \begin{bmatrix} 1 & 3 \\ 2 & 4 \end{bmatrix} =$

(c) $\begin{bmatrix} 4 & 2 \\ -1 & 3 \\ 0 & 5 \end{bmatrix} \begin{bmatrix} -1 & 3 & 0 \\ 4 & -2 & 1 \end{bmatrix} =$

(d) $\begin{bmatrix} -1 & 3 & 0 \\ 4 & -2 & 1 \end{bmatrix} \begin{bmatrix} 4 & 2 \\ -1 & 3 \\ 0 & 5 \end{bmatrix} =$

4. **Linear Transformations**

(a) Prove directly from the definition that the following function is linear:

$$L\left(\begin{bmatrix} x \\ y \\ z \end{bmatrix}\right) = \begin{bmatrix} y + z \\ 2x + y \end{bmatrix}$$

(b) Write a matrix for the linear function.

(c) Is this function one-to-one, onto, both, or neither? Why?

3. **Linear Independence**

(a) *Using the formal definition of linear independence*, show that the set

$$\left\{ \mathbf{u} = \begin{bmatrix} 2 \\ 1 \\ 0 \end{bmatrix}, \mathbf{v} = \begin{bmatrix} -1 \\ 3 \\ 1 \end{bmatrix}, \mathbf{w} = \begin{bmatrix} 1 \\ 0 \\ 2 \end{bmatrix} \right\}$$

is linearly independent.

(b) Determine whether each of the following sets is linearly dependent or linearly independent.

$$\left\{ \begin{bmatrix} -1 \\ 2 \\ 0 \end{bmatrix}, \begin{bmatrix} 2 \\ -1 \\ 1 \end{bmatrix}, \begin{bmatrix} 2 \\ 5 \\ 3 \end{bmatrix} \right\} \qquad \left\{ \begin{bmatrix} 1 \\ 4 \\ 2 \end{bmatrix}, \begin{bmatrix} 3 \\ 1 \\ -1 \end{bmatrix}, \begin{bmatrix} 0 \\ 2 \\ 4 \end{bmatrix} \right\}$$

2. **Vector Equations and Spans**

(a) Is the vector $\mathbf{v} = \begin{bmatrix} 1 \\ 2 \\ 2 \end{bmatrix}$ in the span of the vectors $\mathbf{u}_1 = \begin{bmatrix} 2 \\ 3 \\ 1 \end{bmatrix}, \mathbf{u}_2 = \begin{bmatrix} 3 \\ -1 \\ 1 \end{bmatrix}$?

(b) Write the vector $\mathbf{b} = \begin{bmatrix} 1 \\ 1 \\ 2 \end{bmatrix}$ as a linear combination of the vectors

$$\mathbf{v}_1 = \begin{bmatrix} 1 \\ -1 \\ 2 \end{bmatrix}, \mathbf{v}_2 = \begin{bmatrix} 2 \\ -3 \\ 4 \end{bmatrix}.$$

1. **Systems of Linear Equations**

   Use row reduction to find all solutions to each system of equations.

   (a)

$$\begin{aligned} 2x + y + 3z &= 1 \\ 4x + 3y + 5z &= 1 \\ 6x + 5y + 5z &= -3 \end{aligned}$$

   (b)

$$\begin{aligned} 2x + 3y + z &= 1 \\ x + y + z &= 3 \\ 3x + 4y + 2z &= 4 \end{aligned}$$