Math 2184 §10 Fall 2020
Linear Algebra I Mastery Quiz 6
Due Midnight on Thursday, October 15

This week's mastery quiz has eight topics. **Do not answer all questions.** Please answer the problems on the new topics, labeled 12 and 11. You may answer *one* of the other topics if you did not get a "mastery" grade on it already. If you are retrying a topic you should complete the entire page.

Don't worry if you make a minor error, but try to demonstrate your mastery of the underlying material. You shouldn't spend more than 30 minutes on this quiz. Feel free to consult your notes, but please don't talk about the actual quiz questions with other students in the course.

Remember that you are trying to demonstrate that you understand the concepts involved. For all these problems, justify your answers and explain how you reached them. Do not just write "yes" or "no" or give a single number.

Please upload your work as *one PDF file*. You can produce the file on your computer/tablet/whatever, or you can handwrite it and then scan it. If you have a smartphone, there are many apps that can help you produce a clean single pdf; I personally have used GeniusScan but there are many options.

Topics are:

12. Matrices of Linear Transformations
11. Bases and Coordinates
10. Vector Space Linear Transformations
9. Vector Spaces and Subspaces
8. Basis and Dimension
7. Subspaces
6. Matrix Inverses
4. Linear Transformations

12. **Matrices of Linear Transformations**

(a) Let $L : \mathcal{P}_3(x) \to \mathcal{P}_2(x)$ be defined by $L(f) = 3f'(x) - f'''(x)$. Give a matrix for L with respect to the standard bases $\{1, x, x^2, x^3\}$ and $\{1, x, x^2\}$.

(b) Let $T : \mathbb{R}^3 \to \mathbb{R}^3$ be given by

$$T\left(\begin{bmatrix} x \\ y \\ z \end{bmatrix}\right) = \begin{bmatrix} 3x + 2y - z \\ 2x + 3y + 2z \\ x - 3y + z \end{bmatrix}.$$

Give a matrix for T with respect to the basis $\begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix}$.

11. **Bases and Coordinates**

(a) Prove that $\{1 + x, 3 - x^2, 4x + 2x^2\}$ is a basis for $\mathcal{P}_2(x)$.

(b) Let $B = \left\{ \begin{bmatrix} 1 \\ 4 \\ 2 \end{bmatrix}, \begin{bmatrix} 2 \\ -2 \\ 1 \end{bmatrix}, \begin{bmatrix} 0 \\ 3 \\ -1 \end{bmatrix} \right\}$ be a basis for $\mathbb{R}^3$. Find $\begin{bmatrix} 2 \\ 2 \\ 2 \end{bmatrix}_B$.

10. **Vector Space Linear Transformations**

(a) Let $L : S \to S$ be defined by $L(\{y_k\}) = \{y_k + k\}$, where S is the space of signals. Is L a linear function? Prove your answer.

(b) Let $T : \mathbb{R}^3 \to \mathcal{P}_4(x)$ be defined by

$$T\left(\begin{bmatrix} a \\ b \\ c \end{bmatrix}\right) = ax^4 + (b-c)x^3 + (a+b+c)x.$$

is T a linear function? Prove your answer.

9. **Vector Spaces and Subspaces**

(a) Let $V = M_2 = \left\{ \begin{bmatrix} a & b \\ c & d \end{bmatrix} \right\}$ be the space of 2×2 matrices, and let $U = \left\{ \begin{bmatrix} a & b \\ c & d \end{bmatrix} : a + b + c + d = 1 \right.$
Is U a subspace of V? Prove your answer.

(b) Let $V = M_2 = \left\{ \begin{bmatrix} a & b \\ c & d \end{bmatrix} \right\}$ be the space of 2×2 matrices, and let $W = \left\{ \begin{bmatrix} a & b \\ c & d \end{bmatrix} : a = b + c \right\}$.
Is W a subspace of V? Prove your answer.

8. **Basis and Dimension** Let $A = \begin{bmatrix} 0 & -2 & 1 & 3 \\ 1 & -1 & 2 & 0 \\ 3 & 1 & 1 & 3 \end{bmatrix}$

(a) Find a basis for the nullspace of A.

(b) Find a basis for the columnspace of A.

(c) Find a basis for the rowspace of A.

(d) What is the rank of A?

7. **Subspaces** For each of the following sets, determine whether it is a subspace of $\mathbb{R}^3$, and justify your answer using the definition of subspace.

   (a) $U = \{(x, y, z) : y = 2\}$

   (b) $V = \{(x, y, z) : xy = 0\}$

   (c) $W = \{(x, y, z) : 3x = 5z\}$.

6. **Matrix Inverses**

(a) Find the inverse of $A = \begin{bmatrix} 1 & 2 & 2 \\ 1 & 4 & 1 \\ 0 & 3 & -1 \end{bmatrix}$.

(b) Using part (a), solve the equation $A\mathbf{x} = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$ and $A\mathbf{y} = \begin{bmatrix} 2 \\ 1 \\ 0 \end{bmatrix}$.

4. **Linear Transformations**

(a) Prove directly from the definition that the following function is linear:

$$L\left(\begin{bmatrix} x \\ y \\ z \end{bmatrix}\right) = \begin{bmatrix} x + y + z \\ 2x + y \\ x + 2y + 3z \end{bmatrix}$$

(b) Write a matrix for the linear function.

(c) Is this function one-to-one, onto, both, or neither? Why?