Math 212 Spring 2020
Multivariable Calculus Written HW 2 Solutions
Due Wednesday, February 5

1. Show that $\lim_{(x,y)\to(0,0)} \frac{y^3}{x^2+y^2} = 0$.

   Solution:

$$\left|\frac{y^3}{x^2+y^2} - 0\right| = \frac{y^2}{x^2+y^2}|y| \leq |y| \leq \sqrt{x^2+y^2}.$$

2. Show that the function $\lim_{(x,y)\to(0,0)} \dfrac{x+y}{x-y}$ does not exist. Hint: consider the line $y = mx$.

   Solution:

   Along the line $y = mx$, we have

$$\lim_{(x,y)\to(0,0)} \frac{x+y}{x-y} = \lim_{x\to 0} \frac{x+mx}{x-mx} = \frac{1+m}{1-m}.$$

   Thus the limit depends on the value of m: for instance, if $m = 2$ then teh limit is -3, but if $m = 0$ then the limit is 1. Thus there is no one limit, and so the function doesn't converge.

3. Let $f(x,y) = \dfrac{x^2}{x^2+y}$. Show that along any line $y = mx$ the limit as (x,y) approaches $(0,0)$ exists and is the same. Then use the curve $y = mx^2$ to show that $\lim_{(x,y)\to(0,0)} f(x,y)$ does not exist.

   Solution:

   If $y = mx$ we have

$$\lim_{(x,y)\to(0,0)} f(x,y) = \lim_{x\to 0} \frac{x^2}{x^2+mx} = \lim_{x\to 0} \frac{x}{x+m} = \frac{0}{0+m} = 0$$

   as long as $m \neq 0$.

   But if $y = mx^2$ then

$$\lim_{(x,y)\to(0,0)} f(x,y) = \lim_{x\to 0} \frac{x^2}{x^2+mx^2} = \lim_{x\to 0} \frac{1}{1+m} = \frac{1}{1+m}.$$

   Now the limit depends on m, and for example if $m = 1$ the limit is $\frac{1}{2}$. Thus the limit does not exist.