

Math 212 Spring 2020  
Multivariable Calculus Written HW 3 Solutions  
Due Wednesday, February 12

1. Use the definition of partial derivative to compute  $\frac{\partial}{\partial x}x^2y$ .

**Solution:**

$$\begin{aligned}\frac{\partial}{\partial x}x^2y &= \lim_{h \rightarrow 0} \frac{(x+h)^2y - x^2y}{h} = \lim_{h \rightarrow 0} \frac{x^2y + 2xhy + h^2y - x^2y}{h} \\ &= \lim_{h \rightarrow 0} \frac{2xhy + h^2y}{h} = \lim_{h \rightarrow 0} 2xy + hy = 2xy.\end{aligned}$$

2. Suppose you borrow  $\$A$  at an interest rate of  $r\%$  per month and pay it off over  $t$  months, and the monthly payment is given by some function  $P(A, r, t)$ . For each of the following statements, tell me the units and explain what it tells you about your loan.

(a)  $g(8000, 1, 24) = 376.59$

(b)  $\frac{dg}{dA}(8000, 1, 24) = .047$

(c)  $\frac{dg}{dr}(8000, 1, 24) = 44.83$ .

**Solution:**

- (a) The units are dollars per month. If you borrow 8000 dollars at an interest rate of 1%, and pay it back over 24 months, then your monthly payment is 376.59.
- (b) The units are dollars per month per dollar. This says that if you borrowed one additional dollar, you'd have to pay roughly an extra 4.7 cents per month.
- (c) The units are dollars per month (since percents are unitless); this says that if we increased the interest rate by one percent, you'd have to pay an extra \$44.83 a month.
3. Use difference quotients with  $\Delta x$  and  $\Delta y$  equal to 1 to estimate  $f_x(3, 2)$  and  $f_y(3, 2)$ , where  $f(x, y) = \frac{x^2}{y+1}$ . Then use  $\Delta x = \Delta y = .1$ .

**Solution:** We can estimate

$$\begin{aligned}f_x(3, 2) &\approx \frac{f(4, 2) - f(3, 2)}{1} = \frac{16}{3} - \frac{9}{3} = \frac{7}{3} \\ f_y(3, 2) &\approx \frac{f(3, 3) - f(3, 2)}{1} = \frac{9}{4} - \frac{9}{3} = \frac{-3}{4}.\end{aligned}$$

Taking a smaller interval, we can estimate

$$f_x(3, 2) \approx \frac{f(3.1, 2) - f(3, 2)}{1} = \frac{9.61}{3} - \frac{9}{3} = \frac{.61}{3} = .20333\dots$$
$$f_y(3, 2) \approx \frac{f(3, 2.1) - f(3, 2)}{1} = \frac{9}{3.1} - \frac{9}{3} = \frac{-3}{31} \approx -.097.$$