Math 212 Spring 2020
Multivariable Calculus Written HW 4 Solutions
Due Wednesday, February 19

1. Give a formula for the local linear approximation to $g(x, y, z) = xy + yz + xz$ near the point $(1, 2, 3)$.

   Solution: We have

$$\begin{aligned} g_x(x,y,z) &= y + z & g_x(1,2,3) &= 5 \\ g_y(x,y,z) &= x + z & g_y(1,2,3) &= 4 \\ g_z(x,y,z) &= x + y & g_z(1,2,3) &= 3 \\ & & g(1,2,3) &= 11 \end{aligned}$$

   Thus the linear approximation is

$$g(x, y, z) \approx 11 + 5(x - 1) + 4(y - 2) + 3(z - 3).$$

2. Let $\vec{u} = 3\vec{i} + 4\vec{j}$, and let $f(x, y) = xy$. Compute $f_{\vec{u}}$ directly from the limit definition.

   Solution:

$$\begin{aligned} f_{\vec{u}} &= \lim_{h\to 0} \frac{f(x + 3h/5, y + 4h/5) - f(x, y)}{h} \\ &= \lim_{h\to 0} \frac{(x + 3h/5)(y + 4h/5) - xy}{h} = \lim_{h\to 0} \frac{3yh/5 + 4xh/5 + 12h^2/25}{h} \\ &= \lim_{h\to 0} \frac{3y}{5} + \frac{4x}{5} + 12h25 = \frac{3y}{5} + \frac{4x}{5}. \end{aligned}$$

3. Below is a contour plot of the function $h(x, y)$.

   (a) Sketch the gradient vector at $(0, 1)$.

   (b) Sketch the gradient vector at P.

   (c) Sketch the gradient vector at Q.

   (d) Label a point on the diagram where $\frac{\partial h}{\partial x} = 0$.

   (e) Label a point on the diagram where $\frac{\partial h}{\partial y} = 0$.