

Math 212 Spring 2020  
Multivariable Calculus Written HW 5  
Due Wednesday, February 26

1. Set

$$f(x, y) = \begin{cases} \frac{xy(x^2-y^2)}{x^2+y^2} & (x, y) \neq (0, 0) \\ 0 & (x, y) = (0, 0) \end{cases} .$$

Compute  $f_x(x, y)$  and  $f_y(x, y)$ . Then compute  $\frac{\partial}{\partial y} \frac{\partial}{\partial x} f(0, 0)$  and  $\frac{\partial}{\partial x} \frac{\partial}{\partial y} f(0, 0)$ , and see that they are not the same.

2. Show that the function  $F(x, y) = \arctan(y/x)$  satisfies *Laplace's Equation*  $F_{xx} + F_{yy} = 0$ . (This equation is very important in the theory of differential equations.)
3. Show that the function

$$u(x, t) = \frac{1}{2\sqrt{\pi t}} e^{-x^2/(4t)}$$

satisfies the *heat equation*  $u_t = u_{xx}$  for all  $t > 0$ .