Math 212 Test 3

Instructor: Jay Daigle

Due April 17

- Please try to take only 90 minutes for this test.

  If you want to keep going after 90 minutes, please change pen colors or something and make it clear where the 90 minute end.
- This test is open notes. You may use your notes, and anything I have posted to the website. Please don't use any other sources or materials.
- You may use a normal or scientific calculator. You may not use a graphing calculator. A calculator is not required to complete this exam. Please don't use any interesting computational software or anything that can compute derivatives or integrals for you.
- Read the questions carefully and make sure to answer the actual question asked. Make sure to justify your answers—math is largely about clear communication and argument, so an unjustified answer is much like no answer at all.
- I will do my best to monitor Zoom during the afternoons and evenings to answer questions, but I can't make any promises.
- Good luck!

Name:

Time Started:

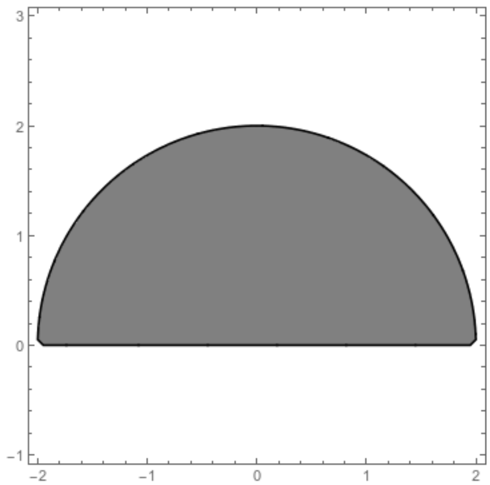
Time Completed:

1	
2	
3	
4	
5	
Σ	

Problem 1. Let $f(x, y) = x^2 + y^2$ and let R be the semicircular region of radius 2 below.

(a) [15 points] Set up integrals to compute $\int_R f\, dA$ in both polar and cartesian coordinates.

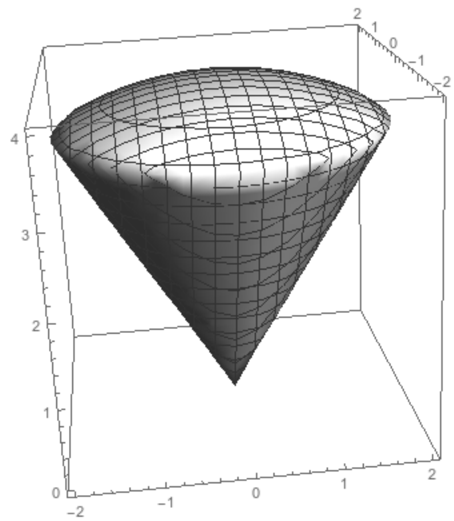
(b) [15 points] Choose one of these integrals and evaluate it.



Problem 2. Let R be the spherical wedge bounded by a sphere of radius 4 centered at the origin, and the cone given by $z = \sqrt{3x^2 + 3y^2}$ (as shown below). Let $f(x, y, z) = z$.

(a) [15 points] Set up integrals to compute $\int_R f\, dA$ in cartesian, cylindrical, and spherical coordinates.

(b) [15 points] Choose one of these integrals and evaluate it.



Problem 3. (a) [10 points] Find a parametric equation for a particle moving in a straight line, starting at $(0, 0)$ and moving towards $(3, 2, 1)$.

(b) [10 points] Suppose another particle follows the path $\vec{r}_2(t) = (t^2, 9 - t, t)$. Does this particle's path intersect the path of the particle from part (a)?

(c) [10 points] Find a parametrization for the cone, opening in the direction of the x axis, with total inner angle $\pi/2$.

Problem 4. (a) [15 points] Compute the integral of the function $f(x) = x + 3y$ over the region bounded by $x + 3y = 0, x + 3y = 3, x - 3y = 0, x - 3y = 2$. (Hint: reparametrize to get a rectangle).

(b) [5 points] Sketch the vector field $\vec{F}(x, y) = x\vec{i} + 1\vec{j}$.

(c) [10 points] Is $\vec{r}(t) = (e^t, t + 1)$ a flow line for $\vec{F}$ from part (b)?

Problem 5. (a) [15 points] Let $f(x, y) = xy$, and let C be the straight line segment from $(2, 2)$ to $(3, 5)$. Compute $\int_C f \, d\vec{r}$.

(b) [15 points] Let $\vec{F}(x, y) = xy\vec{i} + x\vec{j}$, and let C be parametrized by $\vec{r}(t) = (t^2, 3t)$ for $0 \le t \le 2$. Compute $\int_C \vec{F} \cdot d\vec{r}$.