

## Common Notation

Symbol	Meaning	Reference
$\mathbb{R}$	the set of real numbers	1.1
$\in$	is an element of	1.1
$\mathbb{R}^n$	the set of ordered $n$ -tuples	1.1
$\{a, b, d\}$	a set containing $a, b$ , and $d$	1.1
$\{3x : x \in \mathbb{R}\}$	the set of all $3x$ such that $x \in \mathbb{R}$	1.1
$\mathbb{R}^2$	the set of ordered pairs of real numbers; the Cartesian plane	1.1
$\subseteq$	is a subset of	1.1
$\emptyset$	the empty set	1.1
$M_n$	Set of (square) $n \times n$ matrices	1.2
$A^T$	Transpose of $A$	1.4.3
$I_n$	Identity matrix in $M_n$	1.5
$\vec{0}$ or $\mathbf{0}$	the zero vector	1.6, 2.1
$N(A)$ or $\ker(A)$	Nullspace or kernel of matrix $A$	1.6
$\overrightarrow{AB}$	the vector from the point $A$ to the point $B$	2.1
$O$	the point at the origin	2.1
$\vec{v}$ or $\mathbf{v}$	a vector	2.1
$\mathbb{R}^3$	Euclidean threespace	2.2
$V$	vector space	2.3
$\mathcal{P}(x)$	space of polynomials in $x$	2.3
$\mathcal{F}(\mathbb{R}, \mathbb{R})$	the space of functions from $\mathbb{R}$ to $\mathbb{R}$	2.3
$\mathbb{Z}$	the set of integers	2.3
$\cup$	union	3.1
WLOG	Without Loss of Generality	3.2
$\exists$	There exists	
$\mathbf{e}_i$ or $\vec{e}_i$	Standard basis vectors for $\mathbb{R}^n$	3.3

Symbol	Meaning	Reference
$\cong$	Is isomorphic to	4.4
$\sim$	Is similar to	6.2
$\mathbf{u} \cdot \mathbf{v}$	dot product of $\mathbf{u}$ and $\mathbf{v}$	7.1
$\ \mathbf{v}\ $	magnitude of $\mathbf{v}$	7.1
$d(\mathbf{x}, \mathbf{y})$	distance between $\mathbf{x}$ and $\mathbf{y}$	7.1
$\text{proj}_{\mathbf{v}} \mathbf{u}$	The projection of $\mathbf{u}$ onto $\mathbf{v}$	7.1
$\langle \mathbf{u}, \mathbf{v} \rangle$	The inner product of $\mathbf{u}$ and $\mathbf{v}$	7.2
$U^\perp$	Orthogonal complement to $U$	7.4
$\mathbf{v}_U, \mathbf{v}_{U^\perp}$	Orthogonal decomposition	7.4
$\lambda$	Eigenvalue of an operator	5.1
$E_\lambda$	Eigenspace corresponding to the eigenvalue $\lambda$	5.1
$\det A$	Determinant of $A$	5.2
$M_{ij}$	The $i, j$ minor matrix of a matrix $A$	5.2.1
$A_{ij}$	The $i, j$ cofactor of a matrix $A$	5.2.1
$\chi_A(\lambda)$	Characteristic polynomial of $A$	5.3
$\text{Tr}(A)$	Trace of $A$	6.3

# Contents

<b>1</b>	<b>Systems of Linear Equations</b>	<b>2</b>
1.1	Basics of Linear Equations . . . . .	2
1.2	The matrix of a system . . . . .	5
1.3	Row Echelon Form . . . . .	8
1.4	Matrix Algebra . . . . .	13
1.4.1	Simple Operations . . . . .	14
1.4.2	Matrix Multiplication . . . . .	14
1.4.3	Transposes . . . . .	16
1.4.4	Matrices and Systems of Equations . . . . .	17
1.5	The identity matrix and matrix inverses . . . . .	17
1.6	Homogeneous systems and subspaces . . . . .	20
<b>2</b>	<b>Vectors and Vector Spaces</b>	<b>25</b>
2.1	The Cartesian Plane . . . . .	25
2.1.1	Scalar Multiplication . . . . .	26
2.1.2	Vector Addition . . . . .	27
2.2	Threespace and $\mathbb{R}^n$ . . . . .	28
2.3	Vector Spaces . . . . .	31
2.4	Properties of Vector Spaces . . . . .	35
2.5	Subspaces . . . . .	37
<b>3</b>	<b>Spanning sets, linear independence, and bases</b>	<b>42</b>
3.1	Spanning sets . . . . .	42
3.2	Linear Independence . . . . .	53
3.3	Vector Space Bases . . . . .	59
<b>4</b>	<b>Linear Functions</b>	<b>67</b>
4.1	Definition and examples . . . . .	67
4.2	Row space, column space and nullspace . . . . .	70
4.3	The Matrix of a Linear Transformation . . . . .	75
4.4	Isomorphisms . . . . .	83
<b>5</b>	<b>Eigenvectors and Eigenvalues</b>	<b>89</b>
5.1	Eigenvectors . . . . .	89

5.2	Determinants . . . . .	91
5.2.1	The Laplace Formula . . . . .	92
5.2.2	Properties of Determinants . . . . .	94
5.3	Characteristic Polynomials . . . . .	96
<b>6</b>	<b>Similarity and Change of Basis</b>	<b>99</b>
6.1	Change of Basis . . . . .	99
6.2	Similarity . . . . .	103
6.3	Determinant and Trace . . . . .	107
6.4	Diagonalization . . . . .	110
<b>7</b>	<b>Inner Product Spaces and Geometry</b>	<b>117</b>
7.1	The Dot Product . . . . .	117
7.2	Inner Products . . . . .	121
7.3	Orthonormal Bases . . . . .	126
7.4	Orthogonal Subspaces . . . . .	132