Introduction: Changing Perspectives

In this course we want to study "high-dimensional spaces" and "vectors". That's not very specific, though, until we explain exactly what we mean by those things.

An important idea of this course is that it is helpful to study the same things from more than one perspective; sometimes a question that is difficult from one perspective is easy from another, so the ability to have multiple viewpoints and translate between them is extremely useful.

In this course we will take three different perspectives, which I am calling "geometric", "algebraic", and "formal". The first involves spatial reasoning and pictures; the second involves arithmetic and algebraic computations; the third involves formal definitions and properties. The formal perspective is the most abstract and sometimes the most confusing, but often the most fruitful: the formal perspective allows us to take problems that don't look like they involve anything we would call "vectors", and apply the techniques of linear algebra to them anyway.

A common definition of a vector is "something that has size and direction." This is a *geometric* viewpoint, since it calls to mind a picture. We can also view it from an *algebraic* point of view by giving it a set of coordinates. For instance, we can specify a two-dimensional vector by giving a pair of real numbers (x, y), which tells us where the vector points from the origin at $(0, 0)$. From the formal perspective we just have "a vector", which must satisfy certain conditions we'll state later.

In the table below I have several concepts, and ways of thinking about them in each perspective. It's fine if you don't know what some of these things mean, especially in the "formal" column; if you knew all of this already you wouldn't need to take this course.

Geometric	Algebraic	Formal
size and direction	n-tuples	vectors
consecutive motion	pointwise addition	vector addition
stretching, rotations, reflections	matrices	linear functions
number of independent directions	number of coordinates	dimension
plane	system of linear equations	subspace
angle	dot product	inner product
Length	magnitude	norm