

Math 214 Spring 2020  
Linear Algebra HW 1  
Due Thursday, January 30

For all these problems, justify your answers.

1. Find all solutions to the system

$$\begin{aligned}x_1 + 2x_2 - 3x_3 &= 9 \\2x_1 - x_2 + x_3 &= 0 \\4x_1 - x_2 + x_3 &= 4.\end{aligned}$$

2. Find all solutions to the system

$$\begin{aligned}2x_1 + 3x_2 - x_3 + 4x_4 &= 1 \\3x_1 - x_2 + x_4 &= 1 \\3x_1 - 4x_2 + x_3 - x_4 &= 2.\end{aligned}$$

3. Find all solutions to the system

$$\begin{aligned}5x_1 + 3x_2 - x_3 &= 3 \\3x_1 - 3x_2 + x_3 &= 2 \\-x_1 + 2x_2 + 4x_3 &= 1 \\x_1 - x_2 + 3x_3 &= 0.\end{aligned}$$

4. Reduce the following matrix to reduced row echelon form:

$$\begin{bmatrix} 1 & 2 & -4 & -4 & 5 \\ 2 & 4 & 0 & 0 & 2 \\ 2 & 3 & 2 & 1 & 5 \\ -1 & 1 & 3 & 6 & 5 \end{bmatrix}$$

5. Reduce the following matrix to reduced row echelon form:

$$\begin{bmatrix} 2 & -3 & -1 & 2 & 3 & 4 \\ 4 & -4 & -1 & 4 & 11 & 4 \\ 2 & -5 & -2 & 2 & -1 & 9 \\ 0 & 2 & 1 & 0 & 4 & -5 \end{bmatrix}.$$

6. Compute

$$\begin{aligned} & \begin{bmatrix} 6 & 3 & 10 \\ 4 & 8 & 5 \end{bmatrix} \begin{bmatrix} 1 & 1.5 \\ 4 & 3 \\ 1 & 2 \end{bmatrix} = \\ & \begin{bmatrix} 1 & 3 & -1 \\ -2 & -1 & 1 \end{bmatrix} \begin{bmatrix} -4 & 0 & 3 & -1 \\ 5 & -2 & -1 & 1 \\ -1 & 2 & 0 & 6 \end{bmatrix} = \end{aligned}$$

7. Compute:

$$\begin{aligned} & \begin{bmatrix} 1 & 0 & -2 \\ -3 & 1 & 1 \\ 2 & 0 & -1 \end{bmatrix} \begin{bmatrix} 2 & 3 & 0 \\ 1 & -1 & 1 \\ -1 & 6 & 4 \end{bmatrix} = \\ & \begin{bmatrix} 2 & 3 & 0 \\ 1 & -1 & 1 \\ -1 & 6 & 4 \end{bmatrix} \begin{bmatrix} 1 & 0 & -2 \\ -3 & 1 & 1 \\ 2 & 0 & -1 \end{bmatrix} = \end{aligned}$$

8. (★) Let  $A$  be a  $n \times n$  matrix. If  $A$  is invertible, prove that  $A^{-1}$  is unique. In other words, if  $BA = I_n = CA$ , prove that  $B = C$ .

(Hint: what is the matrix  $CAB$  equal to?)