Math 214 Spring 2020
Linear Algebra HW 2
Due Friday, Thursday, February 6

For all these problems, justify your answers.

1. Suppose A is a matrix such that $A^{-1} = \begin{bmatrix} 3 & 1 & 5 \\ 2 & -1 & 5 \\ 1 & 4 & -3 \end{bmatrix}$. Find all solutions to $A\mathbf{x} = \begin{bmatrix} 2 \\ 5 \\ 1 \end{bmatrix}$.
   (Do not try to actually compute the matrix A.)

2. Find the inverse of $\begin{bmatrix} 0 & -1 & 1 & 0 \\ 2 & 1 & 0 & 2 \\ 1 & -2 & 3 & 0 \\ 0 & 1 & 1 & -1 \end{bmatrix}$ or prove it is not invertible.

3. Find the inverse of $\begin{bmatrix} 3 & 2 & 1 & 5 \\ 2 & 4 & 3 & 8 \\ -1 & 2 & 5 & 4 \\ 4 & 8 & 9 & 17 \end{bmatrix}$ or prove it is not invertible.

4. Find the nullspace $\begin{bmatrix} 3 & -2 & 2 & -5 \\ 1 & 0 & -2 & -2 \\ -4 & 2 & -4 & 3 \end{bmatrix}$. (Express your answer as a set).

5. (a) Draw a graph of the Cartesian plane with $\begin{bmatrix} 2 \\ 3 \end{bmatrix}$ and $\begin{bmatrix} -1 \\ 4 \end{bmatrix}$ in standard position.

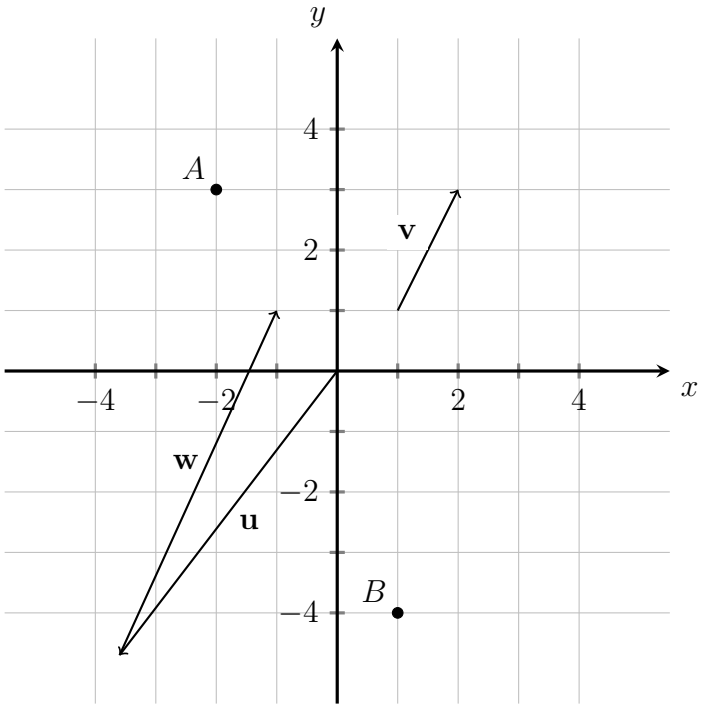
   (b) Draw a graph of the Cartesian plane with the vector $\begin{bmatrix} 3 \\ -1 \end{bmatrix}$ with its tail at the point $(1, 2)$, and the vector $\begin{bmatrix} 2 \\ -4 \end{bmatrix}$ with its tail at $(-1, 3)$.

6. Use the picture below to:

   (a) Write the vector $\overrightarrow{AB}$ in standard vector notation.

   (b) Write the vector $\mathbf{v}$ in standard vector notation.

   (c) Find the vector $\mathbf{u} + \mathbf{w}$ and write it in standard vector notation.



7. (a) If $A = (2, 1)$ and $B = (-2, 2)$, write the vector $\overrightarrow{AB}$ in standard vector notation.

   (b) If $C = (1, -1, 0)$ and $D = (0, 1, 2)$, write the vector $\overrightarrow{CD}$ in standard vector notation.

8. Compute the following:

   (a)

$$\begin{bmatrix} 1 \\ -3/2 \\ 4 \end{bmatrix} + \begin{bmatrix} -7 \\ 2 \\ 1 \end{bmatrix} = \qquad \begin{bmatrix} 1 \\ 5 \\ 3 \\ 7 \\ 2 \end{bmatrix} + \begin{bmatrix} -5 \\ -3 \\ 1 \\ \pi \\ 2 \end{bmatrix} =$$

9. Compute the following:

$$e \cdot \begin{bmatrix} 2 \\ 1 \\ -2 \\ -3 \end{bmatrix} = \qquad -3 \cdot \begin{bmatrix} -7 \\ 3 \\ 1 \end{bmatrix} =$$

10. Let $\mathbf{u} = \begin{bmatrix} -1 \\ 0 \\ 3 \end{bmatrix}$, let $\mathbf{v} = \begin{bmatrix} 4 \\ -2 \\ 7 \end{bmatrix}$, and let $\mathbf{w} = \begin{bmatrix} 0 \\ 5 \\ -3 \end{bmatrix}$.

    (a) Compute $2\mathbf{v} + 3\mathbf{u}$

    (b) Compute $5\mathbf{u} + 2\mathbf{w}$.