Math 214 Spring 2020
Linear Algebra HW 9
Due Thursday, April 16

For all these problems, justify your answers; do not just write "yes" or "no" or give a single number.

1. Let E be the standard basis for $\mathbb{R}^3$, and let $F = \{\mathbf{f}_1, \mathbf{f}_2, \mathbf{f}_3\} = \left\{ \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 2 \\ 2 \end{bmatrix}, \begin{bmatrix} 2 \\ 3 \\ 4 \end{bmatrix} \right\}$.
   (a) Find the transition matrix corresponding to the change of basis from E to F.
   (b) For each of the following vectors (expressed in the standard basis), find the coordinates with respect to F: $(3, 2, 5)$; $(1, 1, 2)$; $(2, 3, 2)$.

2. Let $E = \{\mathbf{e}_1, \mathbf{e}_2, \mathbf{e}_3\} = \left\{ \begin{bmatrix} 4 \\ 6 \\ 7 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 2 \end{bmatrix} \right\}$, and let $F = \{\mathbf{f}_1, \mathbf{f}_2, \mathbf{f}_3\} = \left\{ \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix} \right\}$ be two bases for $\mathbb{R}^3$.
   (a) Find the transition matrix from E to F.
   (b) If $\mathbf{x} = 2\mathbf{e}_1 + 3\mathbf{e}_2 - 4\mathbf{e}_3$, find the coordinates of $\mathbf{x}$ with respect to F.

3. Let
$$L\left(\begin{bmatrix} x \\ y \\ z \end{bmatrix}\right) = \begin{bmatrix} 4x + 2y \\ x + 5y - z \\ x - y + 3z \end{bmatrix}.$$
   Let A be the matrix of L with respect to the standard basis, and let B be the matrix of L with respect to the basis $F = \{(1, 1, 0), (1, 0, 1), (0, 1, 1)\}$.
   (a) Calculate B, the matrix of L with respect to F directly.
   (b) Calculuate B by finding the transition matrix U from F to the standard basis, and calculating $U^{-1}AU$.

4. Compute the traces of the following matrices:
   (a) $\begin{bmatrix} 2 & 5 \\ 1 & 4 \end{bmatrix}$

(b) $\begin{bmatrix} 1 & 1 & 2 \\ 1 & 3 & -1 \\ 4 & 8 & 3 \end{bmatrix}$

(c) $\begin{bmatrix} 1 & 9 & -3 \\ -7 & 2 & -4 \\ 5 & 1 & 1 \end{bmatrix}$

(d) $\begin{bmatrix} 5 & 3 & 7 & 3 \\ 1 & -3 & -8 & 4 \\ 3 & -9 & 2 & 12 \\ 1 & 6 & -11 & 2 \end{bmatrix}$

5. Determine whether each pair of matrices is similar.

(a)
$$A = \begin{bmatrix} 1 & 2 & 4 \\ 3 & 1 & 1 \\ 5 & 2 & 0 \end{bmatrix} \qquad \text{and} \qquad B = \begin{bmatrix} 5 & 2 & 0 \\ -3 & 1 & 1 \\ 4 & 2 & 2 \end{bmatrix}$$

(b)
$$A = \begin{bmatrix} 3 & 5 \\ 4 & 2 \end{bmatrix} \qquad \text{and} \qquad B = \begin{bmatrix} 4 & 5 \\ 4 & 1 \end{bmatrix}$$

(c)
$$A = \begin{bmatrix} 2 & 1 \\ 0 & 2 \end{bmatrix} \qquad \text{and} \qquad B = \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix}$$

(d)
$$A = \begin{bmatrix} 2 & 0 \\ 0 & 1 \end{bmatrix} \qquad \text{and} \qquad B = \begin{bmatrix} 2 & 1 \\ 0 & 1 \end{bmatrix}$$

6. Let S, T be $n \times n$ matrices with $\det S \neq 0$. Let $A = ST$ and $B = TS$. Prove that $A \sim B$.

7. Let $A = \begin{bmatrix} 2 & 3 \\ -1 & -2 \end{bmatrix}$ Diagonalize A and compute A^{10}.

8. Let $B = \begin{bmatrix} 1 & 2 & -1 \\ 2 & 4 & -2 \\ 3 & 6 & -3 \end{bmatrix}$. Diagonalize B and compute B^6.

9. For what real numbers α is the matrix $A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 0 & 0 & \alpha \end{bmatrix}$ not diagonalizable?

10. $(\star)$ Find a matrix B such that $B^2 = A = \begin{bmatrix} 9 & -5 & 3 \\ 0 & 4 & 3 \\ 0 & 0 & 1 \end{bmatrix}$.