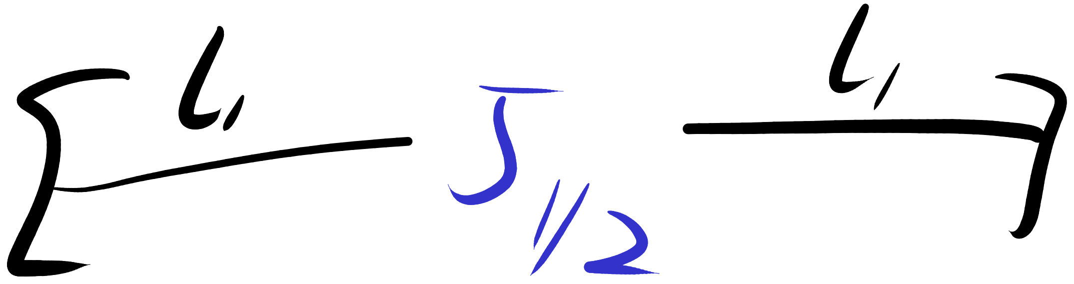
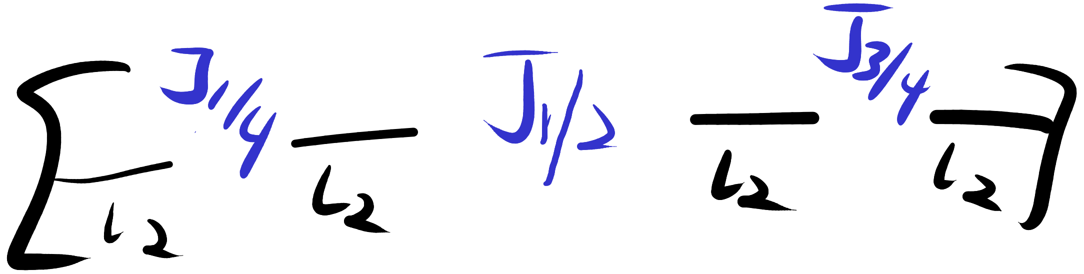
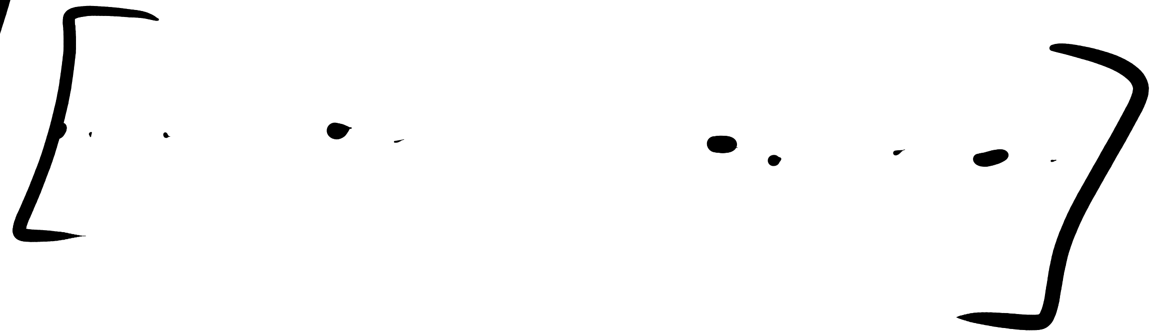
$$1 > 2l_1 > 4l_2 > \cdots > 2^k l_k$$





$$A_j = [0,1] \setminus \bigcup_{\substack{1 \le k \le j \\ 1 \le i \le 2^k - 1}} J_{i/2^k}$$

$$A = [0,1] \setminus \bigcup_{\substack{k \in \mathbb{N} \\ 1 \le i \le 2^k - 1}} J_{i/2^k}$$



A is nowhere dense

A <u>dense</u> in B if $A \subseteq B$ and $\overline{A} = B$.

$\mathbb{Q}$ dense in $\mathbb{R}$.

A <u>nowhere dense</u> if no subset of it is open

A uncountable

A nowhere dense

A closed

$$\lambda(A_K) = 2^K l_K$$

$$\lambda(A) = \lim_{K\to\infty} 2^K l_K$$

$$l_K = \frac{\theta K + 1}{(K+1)2K}$$

$$\lambda(A_K) = \frac{\theta K + 1}{K+1}$$

Suppose $(a,b) \subseteq A$

Then $(a,b) \subseteq A_K$

So $b-a < L_K \to 0$

so $b-a \leq 0$

Lebesgue fn: $f : A^c \to \mathbb{R}$

$$f(x) = \frac{i}{2^k} \quad \text{if } x \in \overline{J}_{i/2^k}$$

increasing

Suppose $|x-y| < l_K$ δ, $x \in J_r$, $y \in J_s$

① $r = s$, then $f(x) = f(y)$

② $r \neq s$ $\quad r, s$ denominators $> 2^K$

$|s-r| < 2^{-K}$ ε

fix $\varepsilon > 0$

find $2^{-K} < \varepsilon$

then if $|x-y| < l_K$

$|f(x) - f(y)| < 2^{-K} < \varepsilon$.

So unif cts on A^c

Exercise: $f: E \to \mathbb{R}$ unif cts on E,

$\exists!$ $F: \overline{E} \to \mathbb{R}$ cts, $F(x) = f(x)$ $\forall x \in E$.

But $\overline{A^c} = [0,1]$

$f(x) = 0 \quad x < 0$

$f(x) = 1 \quad x > 1$

$f_A: \mathbb{R} \to [0,1]$

By IVT, f is surjective

f is non-decreasing

f is al<u>most</u> a bijection from $A \longleftrightarrow [0,1]$

define $B = \{0\} \cup \{\inf J_r\}$

claim $f: A \setminus B \to (0,1)$ is a bijection.

claim: if $x < y$ then $f(x) < f(y)$

if $x, y \in J_r$ for same J_r, then $x = y$.

if $x, y \in A$, $f(x) < f(y)$ unless

<u>$(x, y) = J_r$</u> for some r.

f strictly $\uparrow$ on $A \setminus B$, surjective

so bijection.

C = regular cantor set

$x \in C$ iff $x = \sum_{i=1}^{\infty} \varepsilon_i 3^{-i}$, $\varepsilon_i \in \{0, 2\}$

$$f_C(x) = \sum_{i=1}^{\infty} \varepsilon_i 2^{-i}$$