Math 395 Exam 1

Instructor: Jay Daigle

Problem 1. Prove that If $A_1 \supseteq A_2 \supseteq \ldots$, and further if $\lambda(A_1) < \infty$, then

$$\lambda\left(\bigcap_{k=1}^{\infty} A_k\right) = \lim_{k\to\infty} \lambda(A_k).$$

Problem 2. Let $A = \{x = (x_1, \ldots, x_n) | \exists i : x_i \in \mathbb{Q}\}$ be the set of elements of $\mathbb{R}^n$ with at least one rational coordinate. Prove that $\lambda(A) = 0$.

(Hint: prove that any countable set has measure zero.)

Problem 3. Let X be a set, and $\mathcal{M}_i \subset 2^X$ be a σ-algebra for each i in some index set I. Prove that $\bigcap_{i\in I} \mathcal{M}_i$ is a σ-algebra.

Problem 4. Prove that if $N \subseteq \mathbb{R}^n$ is null, then there is a Borel null set N' such that $N \subseteq N'$. In particular, prove that N' can be chosen to be a G_δ set.