Math 1231 Practice Final

Instructor: Jay Daigle

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- These are the instructions you will see on the real test, next week. I include them here so you know what to expect.
- You will have 120 minutes for this test.
- You are not allowed to consult books or notes during the test, but you may use a one-page, two-sided, handwritten cheat sheet you have made for yourself ahead of time.
- You may use a calculator, but don't use a graphing calculator or anything else that can do symbolic computations. Using a calculator for basic arithmetic is fine.
- This test has fourteen questions, over eleven pages. **You may not answer all questions.**
  - The first five problems are five pages, representing the five major topics, and you should do all five of them. They are worth twenty points each.
  - The remaining nine problems represent the nine secondary topics. You should select **up to four** of these questions and answer them. Your two best will be worth ten points each; the third and fourth will be worth up to five bonus points. It is better to answer two questions well than three or four questions poorly.
  - If you answer more than four out of the last nine problems, we'll ignore the extra, so please pick and choose.
  - If you perform well on a question on this test it will update your mastery scores. Achieving a 18/20 on a major topic or 9/10 on a secondary topic will count as getting a 2 on a mastery quiz.

Name:

Recitation Section:

Problem 1 (M1). (a) Compute $\lim_{x \to 25} \frac{\sqrt{x} - 5}{x - 25}$

(b) Compute $\lim_{x \to 0} \frac{\sin(x^2)}{\sin^2(x)}$

(c) Compute $\lim_{x \to -\infty} \frac{3x}{\sqrt{4x^2 + 3}}$

Problem 2 (M2). (a) Find $\dfrac{d}{dx}\sqrt[4]{\dfrac{x^3+\cos(x^2)}{\sin(x^3)+1}}$

(b) Find a formula for y' in terms of x and y if $x^8+x^4+y^4+y^6=1$.

Problem 3 (M3). (a) If $f(x) = \sqrt{x} + \tan(\pi x)$, use a linear approximation centered at 4 to estimate $f(4.1)$.

(b) A curve is defined by the equation $x^4 - 2x^2y^2 + y^4 = 16$. $(\sqrt{5}, 1)$. What is the equation of the tangent line to the curve at the point $(\sqrt{5}, 1)$?

Problem 4 (M4). (a) Find the absolute extrema of $f(x) = 3x^4 - 20x^3 + 24x^2 + 7$ on $[0, 5]$.

(b) Classify the relative extrema of $g(x) = \dfrac{2x - 1}{x^2 + 2}$.

Problem 5 (M5). (a) Let $G(x) = \int_1^{x^2+1} t\sqrt{1-t^2}\, dt$. What is $G'(x)$?

(b) Compute $\displaystyle\int \sin^4(t)\cos(t)\, dt$

(c) **By explicitly changing the bounds of the integral**, compute $\displaystyle\int_0^4 x^3\sqrt{9+x^2}\, dx$.

Problem 6 (S1)**.** Naming each limit law you use explicitly, compute

$$\lim_{x\to 1} \frac{x^2+4x-5}{x-1}$$

Problem 7 (S2)**.** Use the Squeeze Theorem to show that $\lim_{x\to 5}(x-5)\sin\left(\frac{x^2+1}{x-5}\right) = 0$.

Problem 8 (S3)**. Directly from the definition**, compute $f'(1)$ where $f(x) = \sqrt{x+3}$.

Problem 9 (S4)**.** Suppose that if a car travels at v miles per hour then its fuel efficiency is $F(v) = 8 + 1.3v - .015v^2$ miles per gallon.

(i) What does the derivative $F'(v)$ represent, and what are its units?

(ii) Compute $F'(60)$. What does this tell you?

Problem 10 (S5)**.** A cone with height h and base radius r has volume $\frac{1}{3}\pi r^2 h$. Suppose we have an inverted conical water tank, of height 4m and radius 6m. Water is leaking out of a small hole at the bottom of the tank. If the current water level is 2m and the water level is dropping at $\frac{1}{9\pi}$ meters per minute, what volume of water leaks out every minute?

Problem 11 (S6)**.** Let $j(x) = x^4 - 14x^2 + 24x + 6$. We can compute $j'(x) = 4(x+3)(x-1)(x-2)$ and $j''(x) = 4(3x^2 - 7)$. Sketch a graph of j.

Your answer should discuss the domain, asymptotes, limits at infinity, critical points and values, intervals of increase and decrease, and concavity.

Problem 12 (S7)**.** Use two iterations of Newton's method, starting at 4, to estimate $\sqrt{15}$.

Problem 13 (S8)**.** Using **only the definition of Riemann sum** and your knowledge of limits, compute the exact area under the curve $x^2 + x^3$ between $x = 1$ and $x = 3$.

Problem 14 (S9)**.** What is the centroid (center of mass) of the region bounded by $y = x^3$ and $x = 4$?