Math 1231 Fall 2021
Single-Variable Calculus I Mastery Quiz 1
Due Wednesday, September 15

This week's mastery quiz has two topics. Please do your best on that topic. Don't worry if you make a minor error, but try to demonstrate your mastery of the underlying material. You shouldn't spend more than about 20-30 minutes on this quiz.

Feel free to consult your notes, but please don't talk about the actual quiz questions with other students in the course.

Remember that you are trying to demonstrate that you understand the concepts involved. For all these problems, justify your answers and explain how you reached them. Do not just write "yes" or "no" or give a single number.

Please turn this quiz in at class/recitation on Wednesday. You may print this document out and write on it, or you may submit your work on separate paper; in either case make sure your name and recitation section are clearly on it. If you absolutely cannot turn it in in person, you can submit it electronically but this should be a last resort.

Topics on This Quiz

- Major Topic 1: Computing Limits
- Secondary Topic 1: Definition of a Limit

Name:

Recitation Section:

Major Topic 1: Computing Limits

1. $\lim_{x\to 3} \dfrac{\sqrt{x+1}-2}{x-3} =$

   Solution:

$$\lim_{x\to 3} \frac{\sqrt{x+1}-2}{x-3} = \lim_{x\to 3} \frac{x-3}{(x-3)(\sqrt{x+1}+2)} = \lim_{x\to 3} 1\sqrt{x+1}+2 = \frac{1}{4}$$

2. $\lim_{x\to 2} \dfrac{x^2+x-5}{3-x} =$

   Solution:

$$\lim_{x\to 2} \frac{x^2+x-5}{3-x} = \frac{1}{1} = 1.$$

3. $\lim_{x\to 1} \dfrac{1}{x-1} - \dfrac{1}{x^2-x} =$

   Solution:

$$\lim_{x\to 1} \frac{1}{x-1} - \frac{1}{x^2-x} = \lim_{x\to 1} \frac{x^2-x-(x-1)}{(x-1)(x^2-x)} = \lim_{x\to 1} \frac{(x-1)^2}{x(x-1)^2} = \lim_{x\to 1} \frac{1}{x} = 1.$$

Secondary Topic 1: Definition of a Limit

1. Write a formal ϵ-δ proof that $\lim_{x\to 2} 3x+1 = 7$.

   Solution: Let $\epsilon > 0$ and set $\delta = \epsilon/3$. Then if $0 < |x-2| < \delta$, we have

$$|3x+1-7| = |3x-6| = 3|x-2| < 3\delta = \epsilon.$$

2. Explicitly naming each limit law you use, compute

   $\lim_{x\to 2} 3 \cdot \dfrac{x^2-4}{x-2} =$

   Solution:

$$\begin{aligned}
\lim_{x\to 2} 3 \cdot \frac{x^2-4}{x-2} &= \lim_{x\to 2} 3 \cdot \lim_{x\to 2} \frac{(x-2)(x+2)}{x-2} && \text{Products} \\
&= \lim_{x\to 2} 3 \cdot \lim_{x\to 2} (x+2) && \text{Almost Identical Functions} \\
&= \lim_{x\to 2} 3 \left(\lim_{x\to 2}(x) + \lim_{x\to 2}(2) \right) && \text{additivity} \\
&= 3 \left(\lim_{x\to 2} x + 2 \right) && \text{constants} \\
&= 3(2+2) = 12 && \text{identity.}
\end{aligned}$$