Math 1231 Section 10 Fall 2021
Single-Variable Calculus I Mastery Quiz 11
Due Thursday, December 9

This week's mastery quiz has two topics.

Feel free to consult your notes or speak to me privately, but please don't talk about the actual quiz questions with other students in the course or post about it publicly.

You shouldn't spend more than about 20-30 minutes on this quiz. Don't worry if you make a minor error, but try to demonstrate that you understand the concepts involved and have mastered the underlying material. For all these problems, justify your answers and explain how you reached them. Do not just write "yes" or "no" or give a single number.

Please turn this quiz in class on Monday. You may print this document out and write on it, or you may submit your work on separate paper; in either case make sure your name and recitation section are clearly on it. If you absolutely cannot turn it in in person, you can submit it electronically through Blackboard but this should be a last resort.

Topics on This Quiz

- Major Topic 5: Integration
- Secondary Topic 9: Applications of Integrals

Name:

Recitation Section:

M5: Integration

(a) Compute $\int \cos(5x+3)\,dx =$

Solution: Set $u = 5x+3$ so $du = 5dx$ and $dx = du/5$. Then

$$\int \cos(5x+3)\,dx = \int \cos(u)\frac{du}{5} = \frac{\sin(u)}{5} + C = \frac{1}{5}\sin(5x+3) + C.$$

(b) By explicitly changing the bounds of integration, compute $\int_1^2 x^5\sqrt{x^3+8}\,dx$.

Solution:

We take $u = x^3+8$, so we have $du = 3x^2\,dx, dx = \frac{du}{3x^2}$, and we have $g(1) = 9$ and $g(2) = 16$. Then we compute

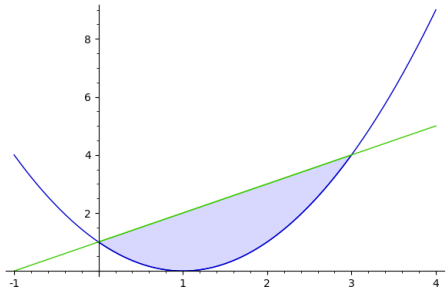
$$\begin{aligned}
\int_1^2 x^5\sqrt{x^3+8}\,dx &= \int_9^{16} x^5\sqrt{u}\frac{du}{3x^2} \\
&= \frac{1}{3}\int_9^{16} x^3\sqrt{u}\,du = \frac{1}{3}\int_9^{16}(u-8)\sqrt{u}\,du \\
&= \frac{1}{3}\int_9^{16} u^{3/2} - 8u^{1/2}\,du \\
&= \frac{1}{3}\left(\frac{u^{5/2}}{5/2} - \frac{8u^{3/2}}{3/2}\right)\Bigg|_9^{16} \\
&= \frac{1}{3}\left(\left(\frac{2\cdot 4^5}{5} - \frac{8\cdot 2\cdot 4^3}{3}\right) - \left(\frac{2\cdot 3^5}{5} - \frac{8\cdot 2\cdot 3^3}{3}\right)\right) \\
&= \frac{2048}{15} - \frac{1024}{9} - \frac{162}{5} + 48 = \frac{1726}{45} = 38.355\ldots.
\end{aligned}$$

S9: Applications of Integrals

(a) Sketch the region bounded by the curves $y = (x-1)^2$ and $y = x+1$. Find the area of the region.

Solution:

We sketch the region, and see that it will be much easier to integrate with respect to x. Setting the two equations equal, we see the curves intersect when $x+1 = x^2-2x+1$, and thus when $0 = x^2-3x = x(x-3)$, and so when $x = 0, 3$. So the x coordinates vary from 0 to 3.



$$\begin{aligned} A &= \int_0^3 x + 1 - (x-1)^2\, dx = \int_0^3 x + 1 - x^2 + 2x - 1\, dx \\ &= \int_0^3 -x^2 + 3x\, dx = -x^3/3 + 3x^2/2\Big|_0^3 \\ &= -9 + 27/2 = 9/2. \end{aligned}$$

(b) A spring with natural length of 30 cm takes 10 N of force to stretch to 40 cm. How much work does it take to stretch it from 40 cm to 50 cm?

Solution: 40 cm is 10cm beyond the natural length, so we have $10 = k \cdot .1$ and thus $k = 100$. Then the work done is

$$W = \int_{.1}^{.2} 100x\, dx = 50x^2|_{.1}^{.2} = 2 - .5 = 1.5J.$$