Math 1231 Section 16 Fall 2021
Single-Variable Calculus I Mastery Quiz 11
Due Monday, November 29

This week's mastery quiz has two topics. You should probably submit both.

Feel free to consult your notes or speak to me privately, but please don't talk about the actual quiz questions with other students in the course or post about it publicly.

You shouldn't spend more than about 20-30 minutes on this quiz. Don't worry if you make a minor error, but try to demonstrate that you understand the concepts involved and have mastered the underlying material. For all these problems, justify your answers and explain how you reached them. Do not just write "yes" or "no" or give a single number.

Please turn this quiz in class on Monday. You may print this document out and write on it, or you may submit your work on separate paper; in either case make sure your name and recitation section are clearly on it. If you absolutely cannot turn it in in person, you can submit it electronically through Blackboard but this should be a last resort.

Topics on This Quiz

- Major Topic 5: Integration
- Secondary Topic 8: Riemann Sums

Name:

Recitation Section:

M5: Integration

(a) Let $F(x) = \int_2^{\sqrt{x^2+1}} t\sin(t)\,dt$. What is $F'(x)$?

Solution: If we set $F_1(x) = \int_2^x t\sin(t)\,dt$ then $F_1'(x) = x\sin(x)$, so

$$\frac{d}{dx}F(x) = \frac{d}{dx}F_1(\sqrt{x^2+1}) = \sqrt{x^2+1}\sin(\sqrt{x^2+1})\frac{2x}{2\sqrt{x^2+1}}.$$

(b) Find $\int \cos(x) + 2x\,dx$.

Solution: $\int \cos(x) + 2x\,dx = \sin(x) + x^2 + C$.

(c) Compute $\int_{-2}^{4} x^3 - 3x\,dx =$

Solution:

$$\begin{aligned}\int_{-2}^{4} x^3 - 3x\,dx &= \frac{x^4}{4} - \frac{3x^2}{2}\Big|_{-2}^{4} \\ &= 64 - 4 - 24 + 6 = 42.\end{aligned}$$

S8: Riemann Sums

Let $f(x) = 2x + 4x^2$ be defined on the interval $[0, 2]$.

(a) Approximate the area under the curve of the function using four rectangles and right endpoints.

(b) Approximate the area under the curve of the function using four rectangles and left endpoints.

(c) Write a formula for R_n, the estimate using n rectangles and right endpoints, as a summation of n terms.

(d) Use your answer in part (c) to find a closed-form formula for R_n. (This formula should not have a summation sign or be given as a sum of n terms.)

(e) Use the formula in part (c) to compute the area exactly.

Solution:

(a) $R_4 = 1/2 \cdot f(1/2) + 1/2 \cdot f(1) + 1/2 \cdot f(3/2) + 1/2 \cdot f(2) = \frac{1}{2}(2 + 6 + 12 + 20) = 20$

(b) $L_4 = 1/2 \cdot f(0) + 1/2 \cdot f(1/2) + 1/2 \cdot f(1) + 1/2 \cdot f(3/2) = \frac{1}{2}(0 + 2 + 6 + 12) = 10$

(c)

$$R_n = \sum_{i=1}^{n} \frac{2}{n} f\left(0 + i\frac{2}{n}\right) = \sum_{i=1}^{n} \frac{2}{n} \left(2(2i/n) + 4(2i/n)^2\right)$$

(d)

$$\begin{aligned}
R_n &= \sum_{i=1}^{n} \frac{2}{n} \left(2(2i/n) + 4(2i/n)^2\right) \\
&= \sum_{i=1}^{n} \frac{2}{n} \left(\frac{4i}{n} + \frac{16i^2}{n^2}\right) \\
&= \sum_{i=1}^{n} \frac{8i}{n^2} + \frac{32i^2}{n^3} \\
&= \frac{8}{n^2} \sum_{i=1}^{n} i + \frac{32}{n^3} \sum_{i=1}^{n} i^2 \\
&= \frac{8}{n^2} \frac{n(n+1)}{2} + \frac{32}{n^3} \frac{n(n+1)(2n+1)}{6}
\end{aligned}$$

(e) We can compute

$$\begin{aligned}
\lim_{n\to+\infty} R_n &= \lim_{n\to+\infty} \frac{8}{n^2} \frac{n(n+1)}{2} + \frac{32}{n^3} \frac{n(n+1)(2n+1)}{6} \\
&= 4 + \frac{32}{3} = \frac{44}{3}.
\end{aligned}$$