Math 1231 Section 10 Fall 2021
Single-Variable Calculus I Mastery Quiz 5
Due Thursday, October 14

This week's mastery quiz has four topics. You may submit all four (but this policy might not be repeated next time there are four topics). If you already have a 4/4 on M1, do not submit M1 this week; if you have a 2/2 on S3, do not submit it.

Feel free to consult your notes or speak to me privately, but please don't talk about the actual quiz questions with other students in the course or post about it publicly.

You shouldn't spend more than about 20-30 minutes on this quiz. Don't worry if you make a minor error, but try to demonstrate that you understand the concepts involved and have mastered the underlying material. For all these problems, justify your answers and explain how you reached them. Do not just write "yes" or "no" or give a single number.

Please turn this quiz in class on Monday. You may print this document out and write on it, or you may submit your work on separate paper; in either case make sure your name and recitation section are clearly on it. If you absolutely cannot turn it in in person, you can submit it electronically through Blackboard but this should be a last resort.

Topics on This Quiz

- Major Topic 2: Computing Derivatives
- Major Topic 3: Linear Approximation
- Secondary Topic 4: Rates of Change

Name:

Recitation Section:

Major Topic 2: Computing Derivatives

Compute the derivative each of the following functions, using any tools we have developed in class.

(a) $\frac{d}{dx}\frac{\sin(\csc(x^2+1))}{x^4+\cos(x)} =$

Solution:

$$\frac{\big(\cos(\csc(x^2+1))(-\csc(x^2+1)\cot(x^2+1))2x\big)(x^4+\cos(x)) - (4x^3-\sin(x))\sin(\csc(x^2+1))}{(x^4+\cos(x))^2}.$$

(b) Find a formula for y' in terms of x and y if $\sqrt{x+y} = x^3y^2$.

Solution:

$$\begin{aligned}
\frac{1}{2}(x+y)^{-1/2}(1+y') &= 3x^2y^2 + 2x^3yy' \\
\frac{1}{2}(x+y)^{-1/2}y' - 2x^3yy' &= 3x^2y^2 - \frac{1}{2}(x+y)^{-1/2} \\
y' &= \frac{3x^2y^2 - \frac{1}{2}(x+y)^{-1/2}}{\frac{1}{2}(x+y)^{-1/2} - 2x^3y}.
\end{aligned}$$

Major Topic 3: Linear Approximation

(a) Give a formula for a linear approximation of $f(x) = x\sqrt{x+1}$ near the point $a = 3$.

Solution:

$$\begin{aligned}
f'(x) &= \sqrt{x+1} + \frac{x}{2\sqrt{x+1}} \\
f'(3) &= 2 + \frac{3}{4} = \frac{11}{4} \\
f(x) &\approx f(a) + f'(a)(x-a) = 6 + \frac{11}{4}(x-3).
\end{aligned}$$

(b) Use your answer in part (a) to estimate $f(3.2)$.

Solution: $f(3.2) \approx 6 + \frac{11}{4}(.2) = 6 + \frac{11}{20} = \frac{131}{20}$.

(c) Write the equation for the tangent line to $g(x) = 2x - \tan(x)$ at the point $a = \pi$.

Solution:

$$\begin{aligned}
g(\pi) &= 2\pi - 0 = \pi \\
g'(x) &= 2 - \sec^2(x) \\
g'(\pi) &= 2 - 1 = 1 \\
y &= 2\pi + (x - \pi)
\end{aligned}$$

Secondary Topic 4: Rates of Change

(a) The *area moment of inertia* of a steel beam measures how difficult it is to bend, and is measured in m^4. If a square beam has a side length of s meters, then its moment of inertia is given by $L(s) = s^4/12$.

(i) What does the derivative $L'(s)$ represent physically, and what are its units?

Solution: $L'(s)$ describes how much increasing the side length by a meter would increase the area moment of inertia. Its units are $\frac{m^4}{m} = m^3$.

(ii) Compute $L'(3)$. What does this tell you physically?

Solution: $L'(s) = 4s^3/12 = s^3/3$ so $L'(3) = 27/3 = 9m^3$. This tells us that if we increase the side length by one meter from 3 meters to 4 meters, we should increase the moment of inertia by about 9 m^4.

(b) Suppose the vertical position of a weight on a spring in inches is given as a function of time in seconds by the formula $h(t) = \cos(3t)$.

(i) When is the velocity zero?

Solution: $p'(t) = -3\sin(3t)$ so the velocity is zero when $\sin(3t) = 0$. This happens when $3t = 0, \pi, 2\pi, \ldots,$ and thus when $t = 0, \pi/3, 2\pi/3, \pi, \ldots$. In other words, at $n\pi/3$.

(ii) When is the acceleration zero?

Solution: $p''(t) = -9\cos(3t)$ is zero when $3t = \pi/2, 3\pi/2, \ldots,$ and thus when $t = \pi/6, 3\pi/6, 5\pi/6, \ldots$. We could say $t = (2n+1)\pi/6$.