Math 1231 Section 16 Fall 2021
Single-Variable Calculus I Mastery Quiz 5
Due Monday, October 11

This week's mastery quiz has three topics. You should submit all three. (It is not possible for you to have completed any of these topics yet.)

Feel free to consult your notes or speak to me privately, but please don't talk about the actual quiz questions with other students in the course or post about it publicly.

You shouldn't spend more than about 20-30 minutes on this quiz. Don't worry if you make a minor error, but try to demonstrate that you understand the concepts involved and have mastered the underlying material. For all these problems, justify your answers and explain how you reached them. Do not just write "yes" or "no" or give a single number.

Please turn this quiz in class on Monday. You may print this document out and write on it, or you may submit your work on separate paper; in either case make sure your name and recitation section are clearly on it. If you absolutely cannot turn it in in person, you can submit it electronically through Blackboard but this should be a last resort.

Topics on This Quiz

- Major Topic 2: Computing Derivatives
- Major Topic 3: Linear Approximation
- Secondary Topic 4: Rates of Change

Name:

Recitation Section:

Major Topic 2: Computing Derivatives

(a) $\dfrac{d}{dx}\sec\left(\dfrac{x^2+1}{\sqrt{x^3-2}}\right) =$

Solution:

$$\sec\left(\frac{x^2+1}{\sqrt{x^3-2}}\right)\tan\left(\frac{x^2+1}{\sqrt{x^3-2}}\right)\frac{2x\sqrt{x^3-2}-(x^2+1)\frac{1}{2}(x^3-2)^{-1/2}\cdot 3x^2}{x^3-2}.$$

(b) $\dfrac{d}{dx}\tan^{3/5}\left(\sec\left(x^3-4\right)\right)$

Solution:

$$\frac{3}{5}\tan^{-2/5}\left(\sec\left(x^3-4\right)\right)\sec^2\left(\sec\left(x^3-4\right)\right) \cdot \sec\left(x^3-4\right)\tan\left(x^3-4\right)3x^2$$

1 Major Topic 3: Linear Approximation

(a) Give a formula for a linear approximation of $f(x) = \frac{x}{x-3}$ near the point $a = 4$.

Solution:

$$\begin{aligned} f'(x) &= \frac{(x-3)-x}{(x-3)^2} \\ f'(4) &= \frac{-3}{1^2} = -3 \\ f(x) &\approx f(a) + f'(a)(x-a) \\ &= 4 - 3(x-4). \end{aligned}$$

(b) Use your answer in part (a) to estimate $f(3.9)$.

Solution: $f(3.9) \approx 4 - 3(-.1) = 4.3$.

(c) Write the equation for the tangent line to $g(x) = \sin(x^2 - 3x)$ at the point $a = 0$.

Solution:

$$\begin{aligned} g'(x) &= \cos(x^2 - x)(2x-3) \\ g'(0) &= 1 \cdot (0-3) = -3 \\ y &= 0 - 3(x-0) \\ &= -3x. \end{aligned}$$

Secondary Topic 4: Rates of Change

(a) Let $F(x) = 1/x + 1$ be the amount of pressure exerted on a beam in pounds per square inch at a point x inches to the right of its left end.

(i) What does the derivative $F'(x)$ represent, and what are its units?

Solution: The derivative $F'(x)$ is the rate at which pressure is increasing as you move to the right along the stick. Its units are pounds per square inch per inch, or pounds per cubic inch.

(ii) Compute $F'(5)$. What does this tell you?

Solution: $F'(x) = -1/x^2$ so $F'(5) = -1/25$. This means that if we are five inches to the right of the endpoint, moving one more inch to the right should decrease the pressure by about 1/25 of a pound per square inch.

(b) Suppose the height of a particle in centimeters is given as a function of time in seconds $p(t) = t^3 - 3t$.

(i) When is the velocity zero?

Solution: $p'(t) = 3t^2 - 3$ is zero when $t = \pm 1$ second.

(ii) When is the acceleration zero?

Solution: $p''(t) = 6t$ is zero when $t = 0$ seconds.