

Math 1231 Section 16 Fall 2021  
Single-Variable Calculus I Mastery Quiz65  
Due Monday, October 25

This week's mastery quiz has four topics. **Submit no more than three.** You may have completed M2 or S4 already, based on your quiz scores. I will try to get the mastery scores from the midterm updated over the weekend, but I don't know when that will happen (and they definitely won't all be uploaded.)

Feel free to consult your notes or speak to me privately, but please don't talk about the actual quiz questions with other students in the course or post about it publicly.

You shouldn't spend more than about 20-30 minutes on this quiz. Don't worry if you make a minor error, but try to demonstrate that you understand the concepts involved and have mastered the underlying material. For all these problems, justify your answers and explain how you reached them. Do not just write "yes" or "no" or give a single number.

Please turn this quiz in class on Monday. You may print this document out and write on it, or you may submit your work on separate paper; in either case make sure your name and recitation section are clearly on it. If you absolutely cannot turn it in in person, you can submit it electronically through Blackboard but this should be a last resort.

**Topics on This Quiz**

- Major Topic 2: Computing Derivatives
- Major Topic 3: Linear Approximation
- Secondary Topic 4: Rates of Change
- Secondary Topic 5: Related Rates

**Name:**

**Recitation Section:**

## Major Topic 2: Computing Derivatives

(a) Find a formula for  $y'$  in terms of  $x$  and  $y$  if  $x^3y + x^2y^2 + y^4 = 0$ .

(b) Compute  $\frac{d}{dx}g(x) = \left(\frac{x \csc(x)}{\sqrt{x^3 - x}}\right)^3$

**Major Topic 3: Linear Approximation**

(a) Estimate  $\sqrt[4]{15}$  using a linear approximation of the function  $\sqrt[4]{x}$  at the point 16.

(b) Find an equation of the line tangent to  $y = \frac{x+1}{x-1}$  at the point  $x = 2$ .

## Secondary Topic 4: Rates of Change

(a) The force a magnet exerts on a piece of iron depends on the distance between the magnet and the metal. Let  $F(d) = \frac{2}{d^2}$  give the force exerted by the magnet in Newtons, where  $d$  is the distance between them in meters.

(i) What does the derivative  $F'(d)$  represent, and what are its units?

(ii) Calculate  $F'(2)$ . What does this tell you physically?

(b) Suppose the distance between two particles in centimeters is given as a function of time in seconds by the formula  $d(t) = t + \frac{1}{t}$ .

(i) When is the velocity zero?

(ii) When is the acceleration zero?

## Secondary Topic 5: Related Rates

A balloon is rising at a constant speed of 5 feet per second. A boy is cycling along a straight road at a speed of 15 feet per second. When he passes under the balloon, it is 45 feet above him. How fast is the distance between the boy and the balloon increasing 3 seconds later? Briefly explain your reasoning.

