Math 1231 Section 10 Fall 2021
Single-Variable Calculus I Mastery Quiz 7
Due Thursday, November 4

This week's mastery quiz has four topics. **You may submit up to three**. This is the last week for topics M2 and S5. M3 is not on this week's quiz, but will reappear on next week's quiz.

Feel free to consult your notes or speak to me privately, but please don't talk about the actual quiz questions with other students in the course or post about it publicly.

You shouldn't spend more than about 20-30 minutes on this quiz. Don't worry if you make a minor error, but try to demonstrate that you understand the concepts involved and have mastered the underlying material. For all these problems, justify your answers and explain how you reached them. Do not just write "yes" or "no" or give a single number.

Please turn this quiz in class on Monday. You may print this document out and write on it, or you may submit your work on separate paper; in either case make sure your name and recitation section are clearly on it. If you absolutely cannot turn it in in person, you can submit it electronically through Blackboard but this should be a last resort.

Topics on This Quiz

- Major Topic 2: Computing Derivatives

- Major Topic 4: Extrema and Optimization

- Secondary Topic 5: Related Rates

- Secondary Topic 6: Curve Sketching

Name:

Recitation Section:

Major Topic 2: Computing Derivatives

Compute the derivative each of the following functions, using any tools we have developed in class.

(a) Compute $\dfrac{d}{dx}\cos^2\left(\tan^2\left(\sec^2(\sqrt{x}+x)\right)\right)$.

(b) Find a formula for $\frac{d^2y}{dx^2}$ in terms of x and y if $\sin(xy) = x + y$.

M4: Extrema and Optimization

(a) Classify the critical points and relative extrema of $g(x) = \dfrac{2x - 1}{x^2 + 2}$.

(b) Find the absolute extrema of $g(x) = x^3 - 3x^2 - 9x + 5$ on $[-2, 4]$, and justify your claim that these are in fact absolute extrema.

S5: Related Rates

A snowball is melting such that its surface area is decreasing at $1\text{cm}^2/\text{min}$. When the radius is 8cm, how quickly is the radius decreasing? Please **write a complete sentence** to answer this question at the end of your work.

(The surface area of a sphere of radius r is $4\pi r^2$.)

S6: Curve Sketching

Let $f(x) = \dfrac{(x-2)^2}{x-1}$. We can compute that

$$f'(x) = \frac{x(x-2)}{(x-1)^2}$$
$$f''(x) = \frac{2}{(x-1)^3}.$$

Sketch a graph of f. Your answer should discuss the domain, roots, asymptotes, limits at infinity, critical points and values, intervals of increase and decrease, and concavity and points of inflection.