Math 2233 Practice Final

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- These are the instructions you will see on the real test, next week. I include them here so you know what to expect.
- This practice test is too long. The real test will be similar but have fewer questions; this is nine pages and I want to write a real final of about six pages. But I wanted to give you more practice, rather than less.
- You will have 120 minutes for the real final.
- You are not allowed to consult books or notes during the test, but you may use a one-page, two-sided, handwritten cheat sheet you have made for yourself ahead of time.
- You may use a calculator, but don't use a graphing calculator or anything else that can do symbolic computations. Using a calculator for basic arithmetic is fine.

Problem 1. (15 points each)

(a) Find a linear approximation of $f(x,y) = \sin(x)\sqrt{1-y^2}$ near the point $(0,0)$. Use it to estimate $f(.1,.1)$.

(b) Find and classify all the critical points of $g(x,y) = x^2 - 3xy + 5x - 2y + 6y^2 + 8$.

(c) Find the minimum value of $f(x,y) = 4xy$ on the unit circle.

Problem 2. (15 points each) Let

$$\vec{F}(x,y,z) = (0,x,y) \qquad \vec{G}(x,y,z) = (2x,z,y) \qquad \vec{H}(x,y,z) = (3y,2x,z).$$

(a) For each field, either find a scalar potential function or prove that none exists.

(b) For each field, either find a vector potential function or prove that none exists.

(c) Let $\vec{r}(t) = (2, 2t, t^2)$. For which of these vector fields is $\vec{r}$ a flow line? Justify your answer.

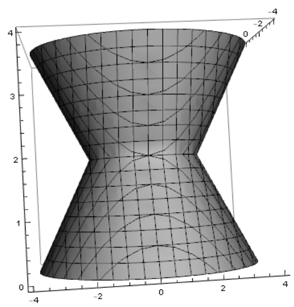
Problem 3. (15 points each) Let $g(x, y, z) = z(x^2 + y^2)$ and let W be a cone with its point at the origin and base given by the circle $z = 2, x^2 + y^2 = 2$.

(a) Set up integrals to compute $\int_W g\,dV$ in cartesian, cylindrical, and spherical coordinates.

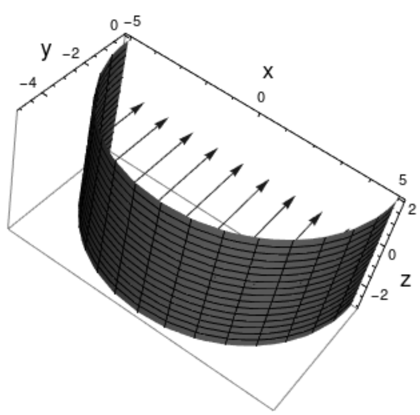
(b) Choose one of the integrals from part (a) and evaluate it.

Problem 4. (10 points each) Set up but **do not evaluate** an integral to answer each of the following questions. Each answer should be an iterated integral containing no vector operations and no variables other than the variables of integration.

(a) Find the volume of the following shape made up of two cones squashed together, which has its base at $z = 0$, its top at $z = 4$, and has a radius of 4 at the base and top, and a radius of 2 at the thinnest point at $z = 2$.



(b) What is the flux of the vector field $\vec{F}(x, y, z) = xy\vec{i} + xz\vec{j} + yz\vec{k}$ through the $y \leq 0$ half of the side of a cylinder of radius 5, centered at the z axis, which goes from $z = -3$ to $z = 2$, oriented towards the z-axis?

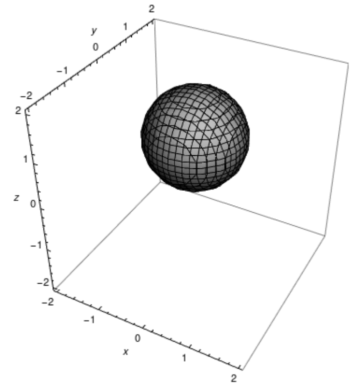


(c) What is the work done by the force field $\vec{G}(x, y, z) = \sin(xz)y\vec{i} + e^{xyz}\vec{j} + \sqrt{x+y+z}\vec{k}$ on a particle following the path $\vec{r}(t) = (t, t^2, t^4)$ from time $t = 0$ to time $t = 5$.

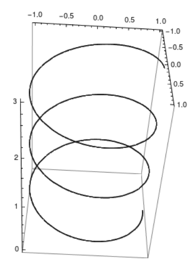
(d) Integrate the function $f(x, y) = 5xy^3$ over the region bounded by $y = 9 - x^2$ and $y = 3 - x$. Sketch the region of integration.

(e) What is the surface area of the graph of $f(x, y) = e^{xy} + \sin(x)\cos(y)$ for $0 \le x \le 3$ and $1 \le y \le \pi$?

(f) Find the mass of a solid spherical ball of radius 1 centered at the point $(0,0,1)$ if its density is given by $\delta(x,y,z) = x^2 z$.

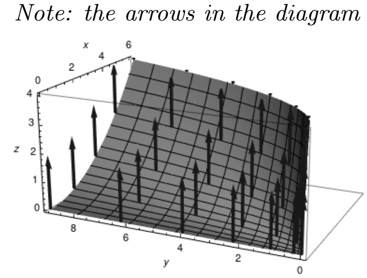


(g) Set up an integral to compute the work done by the force field $\vec{F}(x^2y, yz^3, x+y+z)$ on a particle that moves from $(1,0,0)$ to $(1,0,3)$ by spiraling clockwise around the z-axis three times with radius 1.



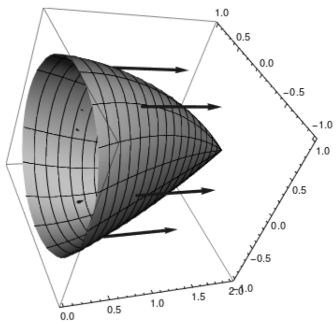
(h) Find the flux of the vector field $\vec{F}(x,y,z) = (x, xy, z)$ through the surface parametrized by $\vec{r}(s,t) = (st, s^2, t^2)$ oriented upwards, for $0 \le s \le 3, 0 \le t \le 2$.

Note: the arrows in the diagram are the orientation of the surface, not a representation of F.

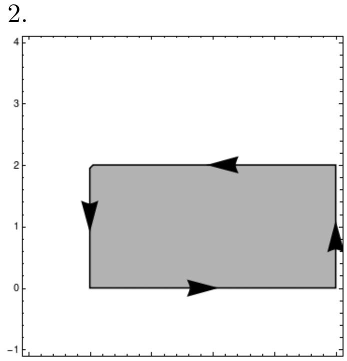


Problem 5. (20 points each) Compute (and evaluate!) each of the following integrals. You may often wish to use a theorem or other result to replace the given integral with an easier integral. Please identify the result you are using.

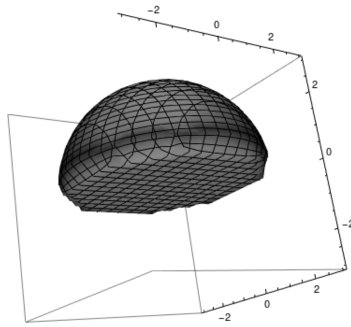
(a) Let $\vec{F}(x, y, z) = \sqrt{x^5 + x}\vec{i} + (x^2yz - z)\vec{j} + (x\sqrt{z^3 + y} + y)\vec{k}$. Compute the flux of the vector field $\nabla \times F$ through a net whose rim is the unit circle $y^2 + z^2 = 1$ in the $x = 0$ plane, oriented in the $\vec{i}$ direction.



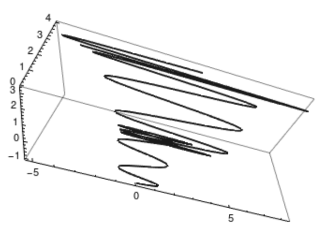
(b) Find the circulation of $\vec{F}(x, y) = -3y\vec{i} + 2x\vec{j}$ counterclockwise around the rectangle $0 \le x \le 4, 0 \le y \le 2$.



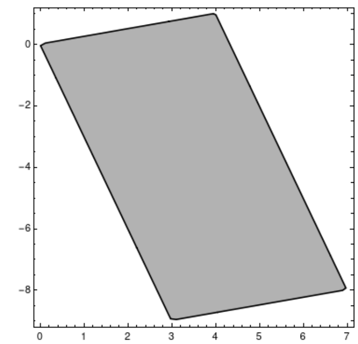
(c) Integrate the function $f(x, y, z) = z$ over the $z \geq 0$ half of the solid radius-3 spherical ball centered at the origin.



(d) Find the integral of the vector field $\vec{F}(x, y, z) = yz\vec{i} + xz\vec{j} + xy\vec{k}$ over the path $\vec{r}(t) = (t + \sin(10\pi t)e^t, t^2 - \cos(2\pi t), 2^t)$ as t varies from 0 to 2.



(e) Compute $\iint_R x + y \, dA$ over the parallelogram with vertices $(0,0), (4,1), (7,-8), (3,-9)$.



(f) Compute $\int_S \vec{F} \cdot d\vec{A}$, where $\vec{F}(x,y,z) = xy^2\vec{i} + x^2y\vec{j} + x^2y^2\vec{k}$ and S is the surface (including both ends!) of a closed cylinder with radius 2 centered on the z-axis, from $z = -2$ to $z = 2$.