

Math 2233 Practice Midterm 1

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Problem 1. Let $f(x, y) = 2xy - x^2y - 2$

- (a) Sketch cross-sections of f for $x = -1, 0, 1$ and $y = -2, 0, 2$.
- (b) If $\vec{u} = \frac{-3}{5}\vec{i} + \frac{4}{5}\vec{j}$, compute $f_{\vec{u}}(2, 1)$.
- (c) At the point $(3, 1)$, in what direction should you move to increase $f(x, y)$ at the fastest possible rate?
What is the rate of increase in that direction?

Problem 2. (a) Give an equation for a plane through the points $(1, 1, 1)$, $(1, 3, 5)$, $(3, 1, -3)$.

(b) Find the cosine of the angle between the vectors $\vec{v} = 3\vec{i} + 2\vec{j} - \vec{k}$ and $\vec{u} = \vec{i} - 2\vec{j} + \vec{k}$.

(c) Let $\vec{v} = 3\vec{i} + \vec{j} - \vec{k}$ and $\vec{u} = -2\vec{i} - \vec{j} + 2\vec{k}$. Compute the orthogonal decomposition of \vec{v} with respect to \vec{u} . That is, write $\vec{v} = \vec{v}_{\text{parallel}} + \vec{v}_{\perp}$.

Problem 3. (a) Compute $\nabla (x^2z + \sqrt{xy})$.

(b) Find an equation for the tangent plane to the graph of the function $f(x, y) = e^{xy} + x/y$ at the point $(0, 2)$.

(c) Let $g(x, y, z) = x^2y + y^2z$. Use a linear approximation at the point $(1, 2, 3)$ to estimate $g(.9, 2.1, 3.2)$.

Problem 4. Let $f(x, y) = e^{xy-4} + x^2y$.

(a) Find the degree 2 Taylor polynomial for f centered at $(2, 2)$.

(b) Use your answer in part (a) to estimate $f(1.9, 2.2)$.

Problem 5. (a) Find and classify the critical points of $f(x, y) = 2x^3 + 6xy + 3y^2$.

(b) Find the critical points of $g(x, y, z) = 9x - 6x^2 + x^3 + x^2yz$.