Math 2233 Fall 2021
Multivariable Calculus Mastery Quiz 10
Due Thursday, December 9

This week's mastery quiz has four topics. **Submit no more than three**. If you already have a 2/2 on a topic, you should not submit it.

Feel free to consult your notes or speak to me privately, but please don't talk about the actual quiz questions with other students in the course or post about it publicly.

Remember that you are trying to demonstrate that you understand the concepts involved. For all these problems, justify your answers and explain how you reached them. Do not just write "yes" or "no" or give a single number.

Please turn this quiz in at class/recitation on Wednesday. You may print this document out and write on it, or you may submit your work on separate paper; in either case make sure your name and recitation section are clearly on it. If you absolutely cannot turn it in in person, you can submit it electronically but this should be a last resort.

Topics on This Quiz

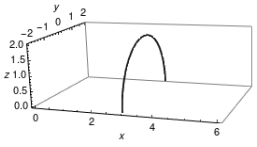
- Topic 11: Line Integrals
- Topic 12: Conservative Vector Fields
- Topic 13: Surface Integrals
- Topic 14: Green's and Stokes's

Name:

Recitation Section:

Topic 11: Line Integrals

(a) Set up, but **do not evaluate**, an integral to compute: the mass of a wire following a semi-circular path of radius 2 contained in the $x = 3$ plane, which goes from $(3, 2, 0)$ through $(3, 0, -2)$ to $(3, -2, 0)$, with density given by $\delta(x, y, z) = x^2 + y^2 + \sqrt{z^2 + 1}$?



(b) Let C be the curve $y = x^2$ from $(0, 0)$ to $(1, 1)$.

Compute the line integral of the vector field $\vec{F}(x, y) = (xy, -x^2)$.

Topic 12: Conservative Vector Fields

(a) Find a potential field for $\vec{F}(x, y, z) = (y + z)\vec{i} + (x + z^2)\vec{j} + (x + 2yz)\vec{k}$ or prove none exists.

(b) Let $f(x, y, z) = x^3y - xz^2$. Compute $\int_C \nabla f \, ds$ where C is parametrized by the curve $\vec{r}(t) = (t + 1, t^2 - 2, \sin(\pi t))$ for $t \in [0, 2]$.

Topic 13: Surface Integrals

(a) The moment of inertia of a surface about the z axis is given by the formula $I = \iint_S x^2 + y^2 \, dS$. Find the moment of inertia of the surface $z = xy$ lying inside the cylinder $x^2 + y^2 = 3$.

(b) Let V be the volume between the spherical shells of radius 1 and 2 centered at the origin. Compute the flux of the vector field $\vec{F}(x, y, z) = x\vec{i} + y\vec{j}$ out of the volume V.

Topic 14: Green's Theorem and Stokes's Theorem

(a) **Use Green's Theorem** to evaluate $\int_C (x\sin(y^2) - y^2)\,dx + (x^2 y\cos(y^2) + 3x)\,dy$ where C is the counterclockwise boundary of the trapezoid with vertices $(0,-2), (1,-1), (1,1), (0,2)$.

(b) **Use Stokes's Theorem** to compute $\iint_S \nabla \times \vec{F} \cdot d\vec{S}$ where $\vec{F}(x,y,z) = (z-y, x, -x)$ and S is the hemisphere $x^2 + y^2 + z^2 = 4, z \geq 0$, oriented inwards towards the center of the hemisphere.