Math 2233 Fall 2021
Multivariable Calculus Mastery Quiz 4
Due Thursday, October 14

This week's mastery quiz has five topics. **Submit no more than three**. If you already have a 2/2 on a topic, you should not submit it. Please **check Blackboard for updated scores**, since your midterm performance can impact your mastery score. You may only need to submit topic 6; please do submit topic 6 regardless of your other choices. This week will be the last week Topics 2 and 3 are on the quiz.

Feel free to consult your notes or speak to me privately, but please don't talk about the actual quiz questions with other students in the course or post about it publicly.

Remember that you are trying to demonstrate that you understand the concepts involved. For all these problems, justify your answers and explain how you reached them. Do not just write "yes" or "no" or give a single number.

Please turn this quiz in at class/recitation on Wednesday. You may print this document out and write on it, or you may submit your work on separate paper; in either case make sure your name and recitation section are clearly on it. If you absolutely cannot turn it in in person, you can submit it electronically but this should be a last resort.

Topics on This Quiz

- Topic 2: Vector Operations
- Topic 3: Partial Derivatives and Linear Approximation
- Topic 4: Gradient and Directional Derivatives
- Topic 5: Multivariable Optimization
- Topic 6: Constrained Optimization

Name:

Recitation Section:

Topic 2: Vector Operations

(a) Find the orthogonal decomposition of $\vec{v} = 6\vec{i} + 2\vec{j} - 3\vec{k}$ with repsect to $\vec{u} = 2\vec{i} + \vec{j} + 2\vec{k}$.

(b) Find the area of the parallelogram with vertices $(0, 0, 0), (2, 2, 2), (1, 5, 5), (3, 7, 7)$.

(c) Find $\cos\theta$ where θ is the angle between $\vec{u} = \vec{i} + 2\vec{j} + 3\vec{k}$ and $\vec{v} = \vec{i} - \vec{j} + \vec{k}$.

Topic 3: Partial Derivatives and Linear Approximation

(a) Give an equation for the plane tangent to $f(x, y) = x \sin(xy)$ at the point $(1, \pi)$.

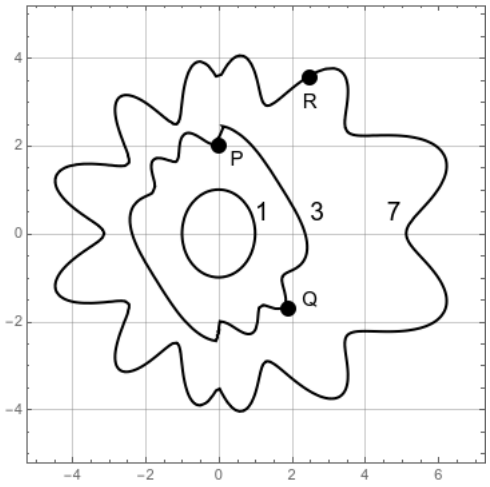
(b) Set $g(x, y) = ye^{2x+y}$. Use a linear approximation to estimate $g(-.9, 2.1)$.

(c) Let $h(x, y, z) = \tan(x^2y + y^2z) - xz$. Compute $\nabla h(x, y, z)$.

Topic 4: Gradients and Directional Derivatives

(a) Below is a contour plot of the function $h(x, y)$.

- Sketch the gradient vector at R.
- Estimate $\frac{\partial h}{\partial y}$ at the point P. Explain your reasoning in a sentence or so.



(b) Let $f(x, y, z) = \ln(xy + z)$. Find the directional derivative in the direction $-\vec{i} - \vec{j} + \vec{k}$ at the point $(0, 3, 1)$.

(c) Find all three second partial derivatives of $g(x, y) = x^2y + xy^3$.

Topic 5: Multivariable Optimization

(a) Find and classify the critical points of $f(x, y) = (3x + 4x^3)(y^2 + 2y)$.

(b) Find (but don't classify) the critical points of $g(x, y, z) = x^2y - xz^2$.

Topic 6: Constrained Optimization

(a) Find the maximum and minimum values of $f(x, y) = xy$ subject to the constraint that $x^2 + 4y^2 \leq 1$.

(b) Find the maximum and minimum values of $g(x, y, z) = y^2 - 10z$ subject to the constraint $x^2 + y^2 + z^2 = 36$.