Math 2233 Fall 2021
Multivariable Calculus Mastery Quiz 6
Due Thursday, October 28

This week's mastery quiz has five topics. **Submit no more than four**. If you already have a 2/2 on a topic, you should not submit it. Please **check Blackboard for updated scores**, since your midterm performance can impact your mastery score. You may only need to submit topics 8 and 9. This week will be the last week Topics 5 and 6 are on the quiz.

Feel free to consult your notes or speak to me privately, but please don't talk about the actual quiz questions with other students in the course or post about it publicly.

Remember that you are trying to demonstrate that you understand the concepts involved. For all these problems, justify your answers and explain how you reached them. Do not just write "yes" or "no" or give a single number.

Please turn this quiz in at class/recitation on Wednesday. You may print this document out and write on it, or you may submit your work on separate paper; in either case make sure your name and recitation section are clearly on it. If you absolutely cannot turn it in in person, you can submit it electronically but this should be a last resort.

Topics on This Quiz

- Topic 5: Multivariable Optimization
- Topic 6: Constrained Optimization
- Topic 7: Multivariable Integrals
- Topic 8: Integrals in Other Coordinates
- Topic 9: Calculus of Curves

Name:

Recitation Section:

Topic 5: Multivariable Optimization

(a) Find and classify the critical points of $f(x, y) = 2x^3 - 6xy + y^2$.

(b) Find (but don't classify) the critical points of $g(x, y, z) = x^3 + y^3 - 3x^2 - y^2 - z^2 + 2z - 1$.

Topic 6: Constrained Optimization

Use the method of Lagrange multipliers to find the point on the circle $x^2 + y^2 = 40$ closest to the point $(1, 3)$.

Topic 7: Multivariable integrals

(a) Sketch the region of integration and compute $\iint_R xy^2\,dx\,dy$, where R is the region in the first quadrant bounded by the curves $y = x^2$ and $x = y^2$. (Do not use a calculator!)

(b) Find the mass of the tetrahedron bounded by the planes $x = 0, y = 0, z = 0$, and $x + 2y + 3z = 6$ if the density is given by $\delta(x, y, z) = z$.

Topic 8: Integrals in Other Coordinate Systems

(a) Sketch the region of integration and compute $\iint_R y\sqrt{x^2+y^2}\,dA$ where R is the region given by $x^2+y^2 \leq 4$ and $0 \leq y \leq x$.

(b) Find the volume of the solid bounded by the paraboloid $z = x^2+y^2$ and the sphere $x^2+y^2+z^2 = 6$ (above the plane z=0).

Topic 9: Calculus of Curves

(a) Find a parametric equation for a particle moving in a straight line, starting at $(0, 0, 0)$ and moving towards $(3, 2, 1)$.

(b) Suppose another particle follows the path $\vec{r}_2(t) = (t^2, 9-t, t)$. Does this particle's path intersect the path of the particle from part (a)?

(c) Find an equation for the line tangent to the curve $\vec{r}(t) = (3t, \ln(t^2+1), 5t^2+2)$ at the time $t = 3$.