Math 2233 Fall 2021
Multivariable Calculus Mastery Quiz 9
Due Thursday, December 2

This week's mastery quiz has five topics. **Submit no more than four**. If you already have a 2/2 on a topic, you should not submit it. This week will be the last week Topic 10 is on the quiz.

Feel free to consult your notes or speak to me privately, but please don't talk about the actual quiz questions with other students in the course or post about it publicly.

Remember that you are trying to demonstrate that you understand the concepts involved. For all these problems, justify your answers and explain how you reached them. Do not just write "yes" or "no" or give a single number.

Please turn this quiz in at class/recitation on Wednesday. You may print this document out and write on it, or you may submit your work on separate paper; in either case make sure your name and recitation section are clearly on it. If you absolutely cannot turn it in in person, you can submit it electronically but this should be a last resort.

Topics on This Quiz

- Topic 10: Change of Variables
- Topic 11: Line Integrals
- Topic 12: Conservative Vector Fields
- Topic 13: Surface Integrals
- Topic 14: Green's and Stokes's

Name:

Recitation Section:

Topic 10: Change of Variables

Use a change of variables to integrate the function $(x+y)(x-y)$ over the diamond bounded by $x+y=2, x+y=0, x-y=0, x-y=2$.

Topic 11: Line Integrals

(a) Compute the arc length of the curve $\vec{r}(t) = (t^2, 3, \frac{1}{3}t^3)$ between the points $(0, 3, 0)$ and $(1, 3, 1/3)$.

(b) Let C be the helix that winds around the cylinder $x^2 + y^2 = 1$ (counterclockwise viewed from the positive z-axis looking down on the xy-plane), starting at $(1, 0, 0)$, winding around the cylinder once, and ending at the point $(1, 0, 1)$.

Compute the line integral of the vector field $\vec{F}(x, y, z) = (-y, x, z^2)$. (Hint: this vector field is **not** conservative.)

Topic 12: Conservative Vector Fields

(a) Find a potential field for $\vec{F}(x,y,z) = y\vec{i} + x\vec{j} + xz\vec{k}$ or prove none exists.

(b) Let $f(x,y,z) = 3x^2y + \frac{z}{x}$. Compute $\int_C \nabla f \, ds$ where C is parametrized by the curve $\vec{r}(t) = (t, \frac{t^2}{t+4}, t)$ for $t \in [1,4]$.

Topic 13: Surface Integrals

(a) Find the surface area of the part of the paraboloid $z = 16 - x^2 - y^2$ which lies above the xy-plane.

(b) Compute the flux of the vector field $\vec{F}(x, y, z) = y\vec{i} + z\vec{k}$ upwards through portion of the cone $z = 1 - \sqrt{x^2 + y^2}$ above the plane $z = 0$.

Topic 14: Green's Theorem and Stokes's Theorem

(a) **Use Green's Theorem** to evalute $\int_C \vec{F} \cdot d\vec{r}$ where $\vec{F}(x,y) = x^2y^2\vec{i} + 2xy\vec{j}$ and C is the boundary of the square (clockwise) through points $(0,0), (0,3), (3,3), (3,0)$.

(b) **Use Stokes's Theorem** to compute $\int_C \vec{F} \cdot d\vec{r}$ where $\vec{F}(x,y,z) = (x^2 - y, y^2 + x, 1)$ and C is the intersection of $x^2 + y^2 = 1$ and $z = y^2$ viewed from the point $(0,0,100)$.