§1 Transendental Functions

§1.1 Invertible functions

A function is a <u>rule</u> that assigns one output to each input.

Can we <u>undo</u> a fn?

Dfn: f a fn
$g(f(x)) = x$ for every x,
then g is an inverse for f.

- $f(x) = x$ then $g(y) = y$ is an inverse.

- $f(x) = 5x+3$

if $f(x) = 13$
then $x = 2$

$13 = 5x + 3$

$y = 5x+3$

$y - 3 = 5x$

$\frac{y-3}{5} = x$

$g(y) = \frac{y-3}{5}$

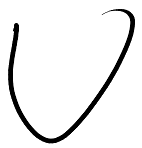
$g(8) = \frac{8-3}{5} = 1.$

- $f(x) = x^3$ $\quad g(y) = \sqrt[3]{y}$

- f has at most one inverse

- suppose $f(x) = x^2$

  $f(a) = 9.$



  $a = 3$ <u>or</u> $a = -3$

  <u>No inverse!</u>

- $f(x) = \sin(x)$

  $f(a) = 0.$

  $a = 0, \pi, 2\pi, -\pi, -2\pi$

  ∞ inputs for this output

  No inverse

Dfn: f is 1-1 (one-to-one, injective, a monomorphism, monic) if, whenever $f(a) = f(b)$, then $a = b$.

Examples:

- $f(x) = x$

  If $f(a) = f(b)$, then $a = b$. Yes 1-1

- $f(x) = 5x + 3$

  If $f(a) = f(b)$, then $5a + 3 = 5b + 3$

  so $5a = 5b$, so $a = b$. Yes

- $f(x) = |x|$

  $|1| = |-1|$, so $f(1) = f(-1)$, but $1 \neq -1$.

  No, not 1-1.

$f(x) = x^2$ No

$(-3)^2 = 3^2$

if $a^2 = b^2$

$|a| = \sqrt{a^2} = \sqrt{b^2} = |b|$

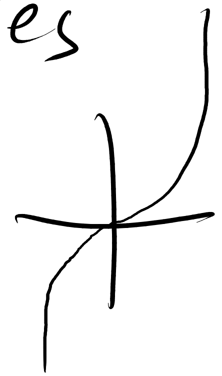
$f(x) = x^3$ Yes

if $a^3 = b^3$

$\sqrt[3]{a^3} = \sqrt[3]{b^3}$

$\quad \| \qquad\quad \|$

$\quad a \qquad\quad b$

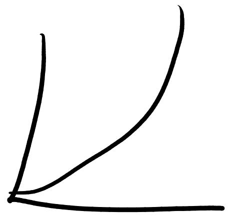


$f(x) = 7$ (domain = reals)

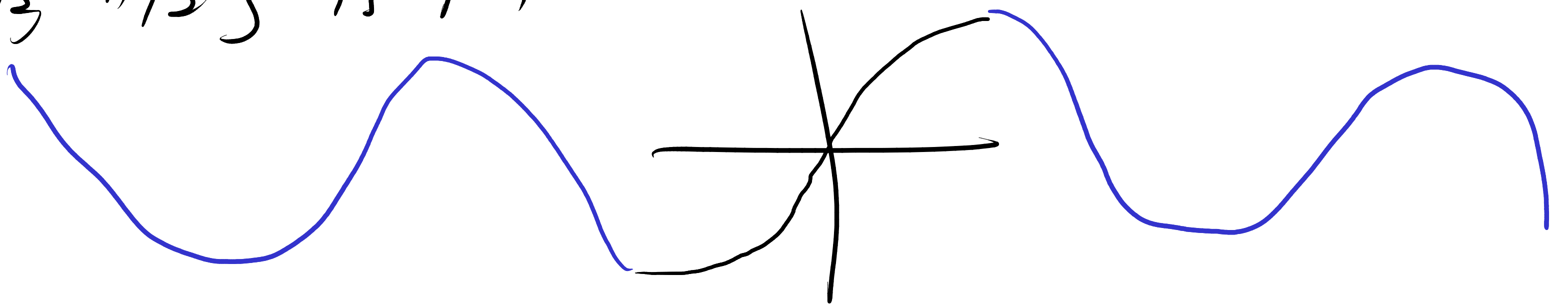
no

Some functions not 1-1, but we can fix them.

$f(x) = x^2$ for $x \geq 0$ is 1-1.



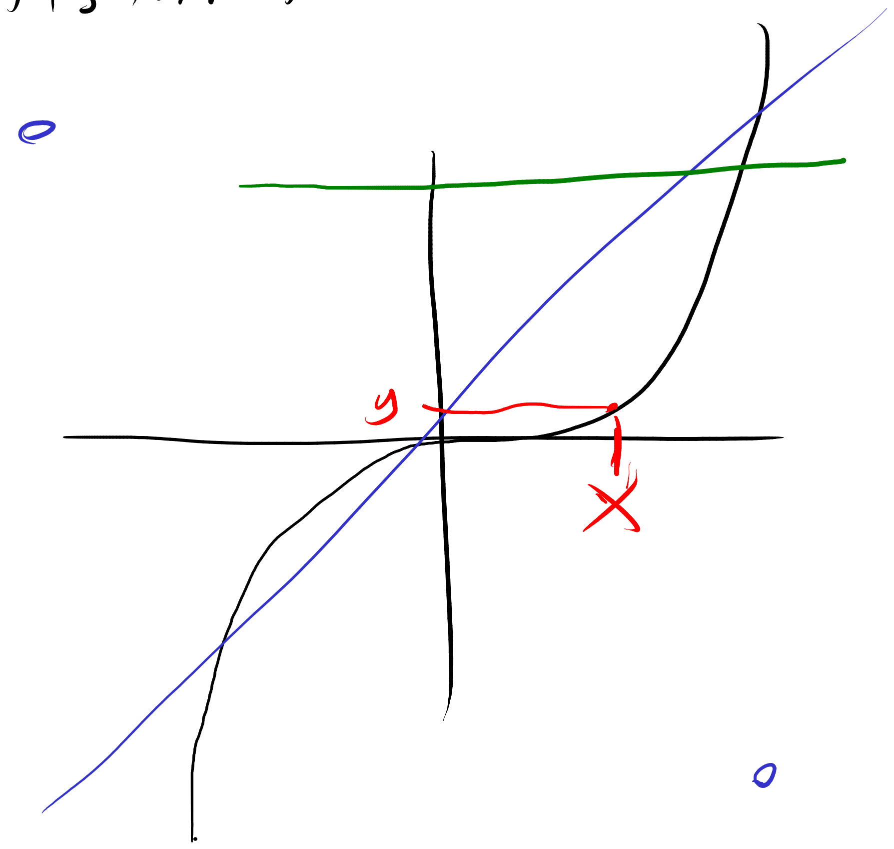
$\sin(x)$ on $[-\pi/2, \pi/2]$ is 1-1



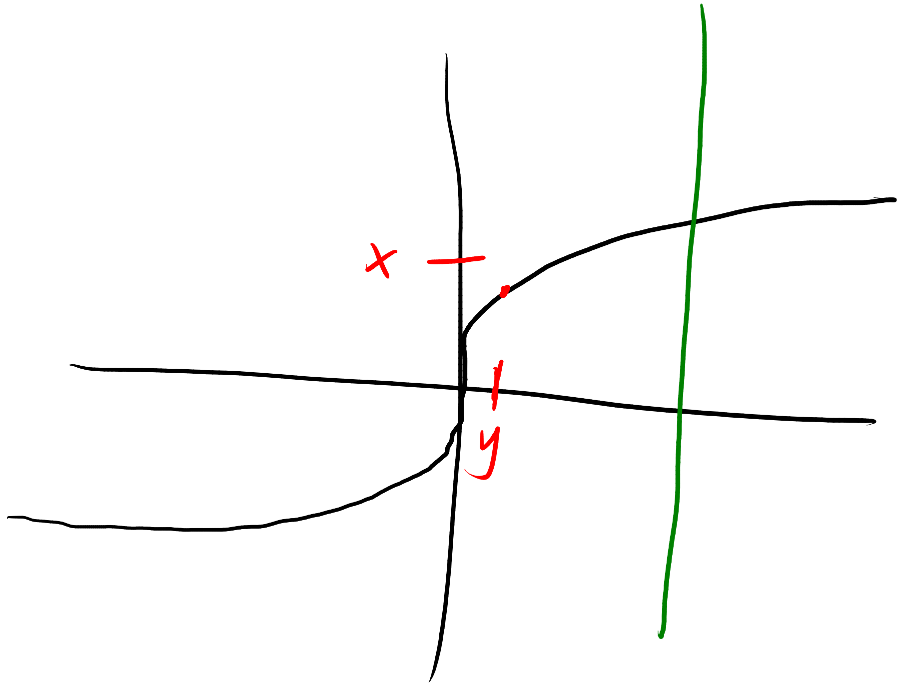
Proposition (Horizontal line test):

f is 1-1 if and only if any horizontal line intersects graph at at most one point.

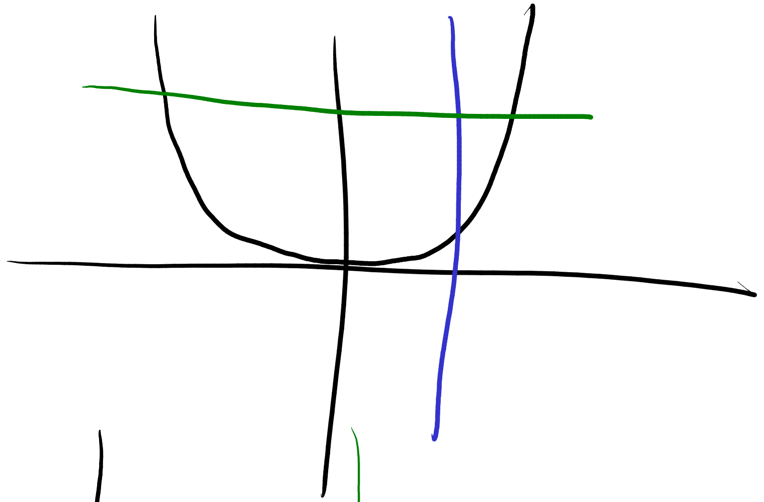
a function is 1-1
if and only if
it is invertible

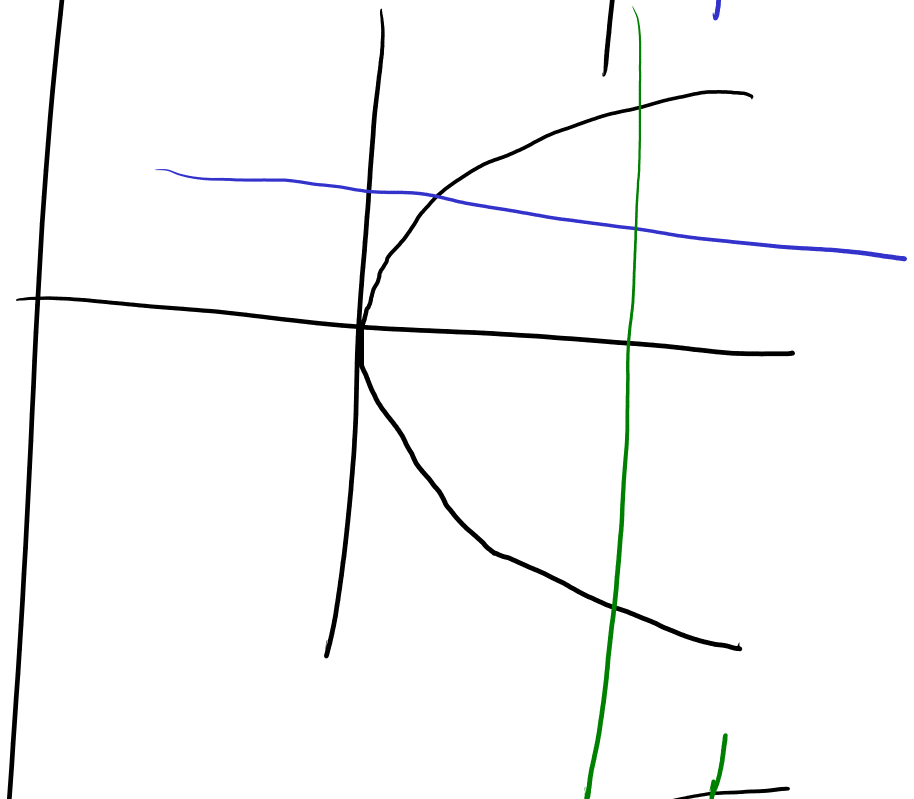


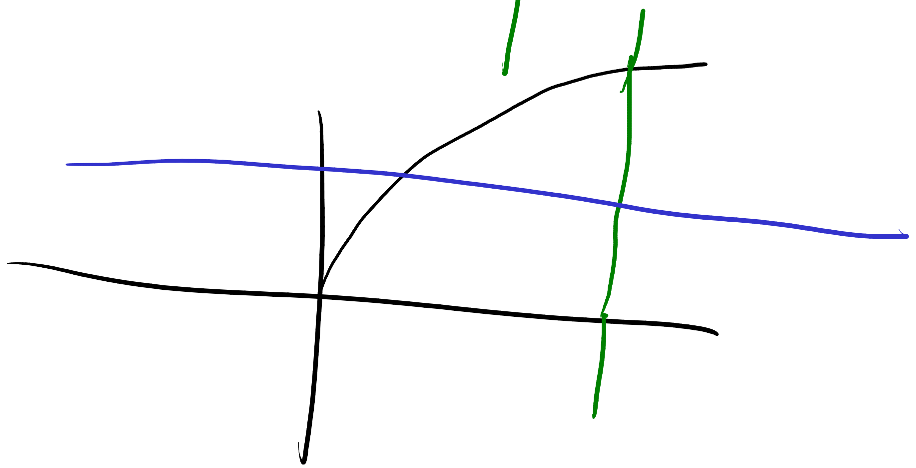
x^3



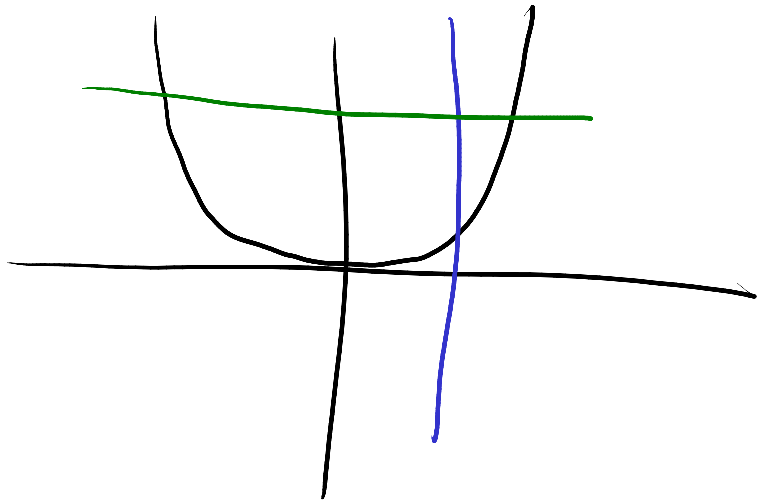
$\sqrt[3]{x}$





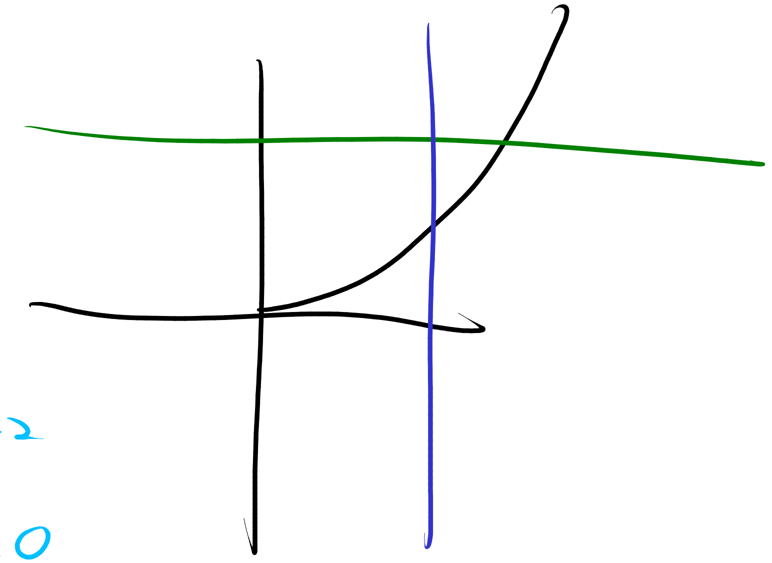


x^2



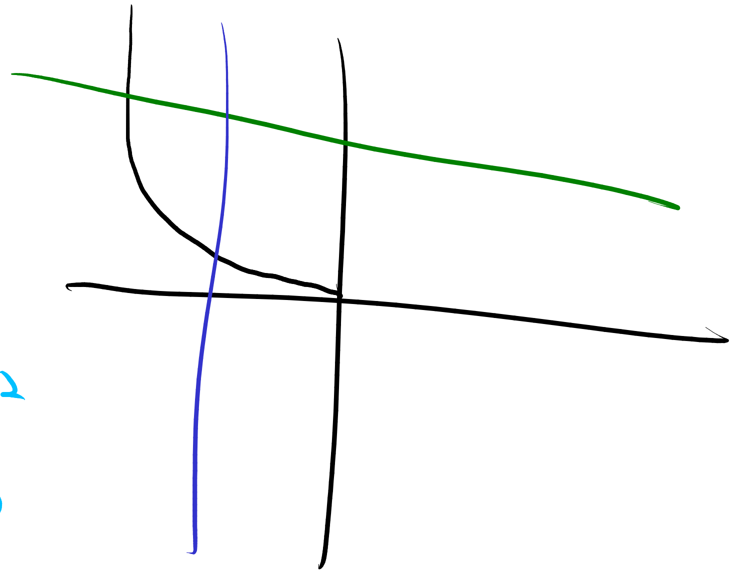
x^2

$x \geq 0$

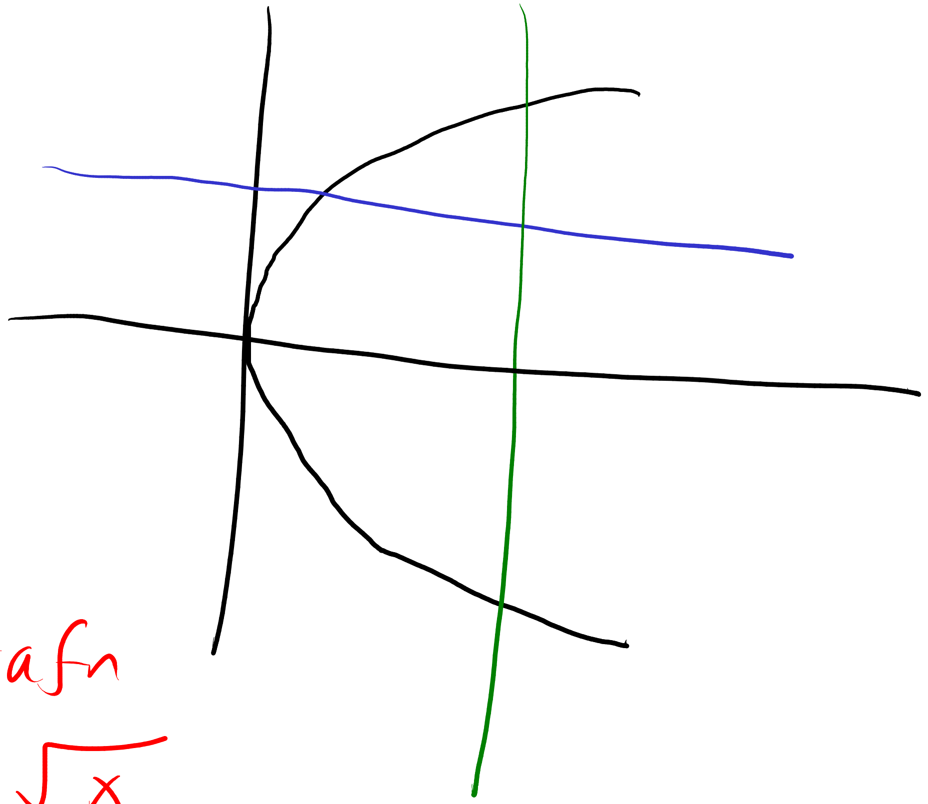


x^2

$x \leq 0$

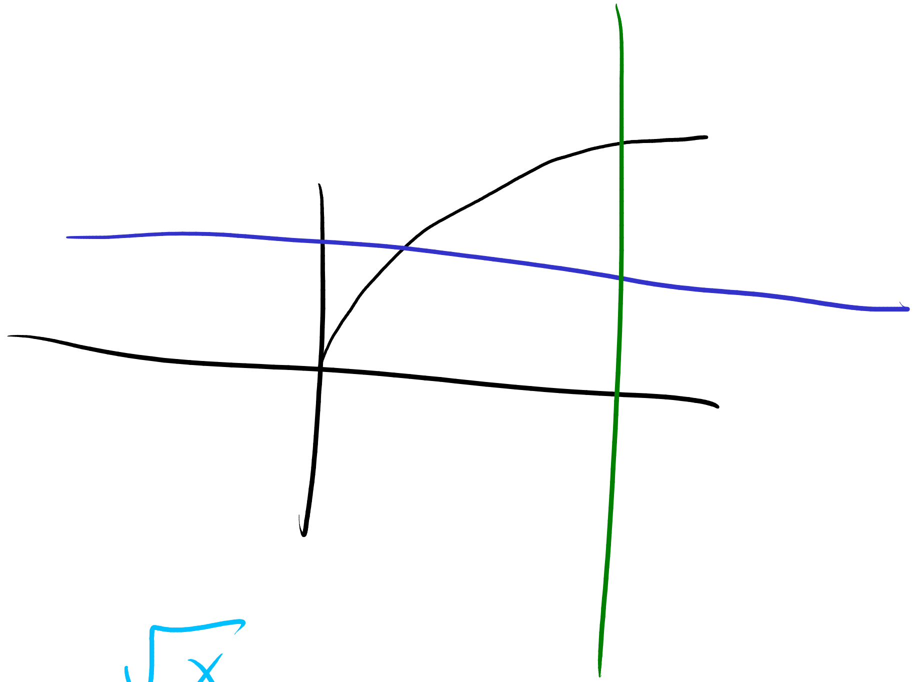


neg → pos

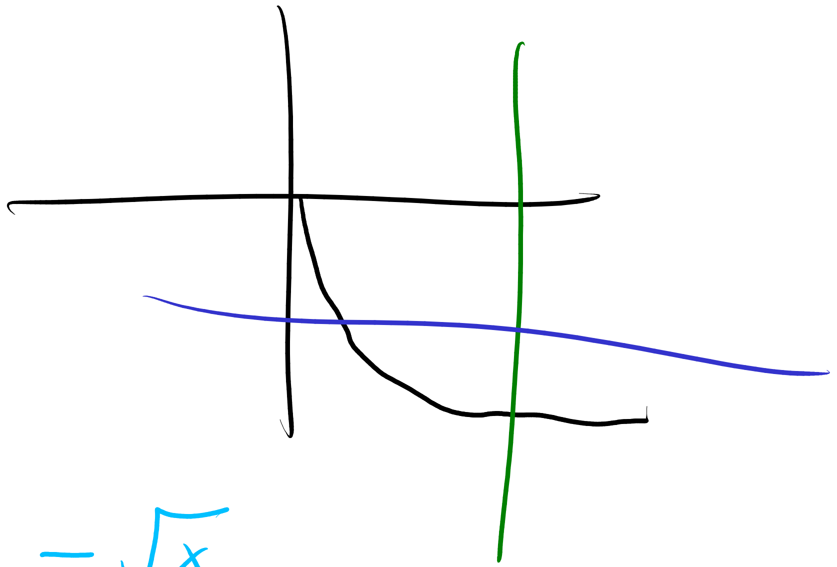


not a fn

$\pm\sqrt{x}$



$\sqrt{x}$



$-\sqrt{x}$

pos → neg

The inverse of f is denoted f^{-1}

Fact: If f is continuous f^{-1} is continuous

If f is cts at a, then f^{-1} is cts at $f(a)$.

suppose $y = f(x)$

$$\frac{dy}{dx} = \frac{1}{\frac{dx}{dy}}$$

$$f^{-1}(f(x)) = x$$

$$(f^{-1})'(f(x)) \cdot f'(x) = 1$$

$$\Rightarrow (f^{-1})'(f(x)) = \frac{1}{f'(x)}$$

$$\Rightarrow (f^{-1})'(y) = \frac{1}{f'(f^{-1}(y))}.$$

set $y = f(x)$

$x = f^{-1}(y)$