Differential Equations

. An eqn relates derivatives of f to f.

"Acceleration is proportional to force"

$$F = ma$$

what if force ~ position

$$F(x) = -kx$$

(Hooke's Law)

$$F(t) = -kp(t)$$

$$-kp(t) = ma(t) = m v'(t) = m p''(t)$$

$$\Rightarrow p''(t) = -\frac{k}{m} p(t)$$

Differential eqn

Assume $k = m$

$$p''(t) = -p(t)$$

$\cos(x)$, $\sin(x)$, $-\sin(x)$, 0

simple harmonic motion

$$p(t) = A\cos(t) + B\sin(t)$$

$y' = f(x)$

$\Rightarrow y = \int f(x)\,dx$

$x = 3t5$

Proportional Growth

$y' = ky$

doubling every year: $k = 1$

100% growth

$y = Ce^{kx}$

$\Rightarrow y' = ke^{kx} = ky$

$y = 3e^{kx}$

$y' = 3ke^{kx} = ky$

economic growth

interest rates

radioactive decay

Heat transfer

epidemiology

Rule of 70

time to double \$ $\approx \dfrac{70}{\text{interest rate}}$

$y = Ce^{rt}$

for what t is $y(t) = 2y(0)$?

$\cancel{C}e^{rt} = 2\cancel{C}e^{0}$

$e^{rt} = 2$

$rt = \ln(2)$

$t = \dfrac{\ln(2)}{r} \approx \dfrac{.7}{r}$

Evans price change model

$$\frac{dp}{dt} = k(D-S) = k(a - bp - r - sp)$$
$$= k(a-r) - k(s+b)p$$

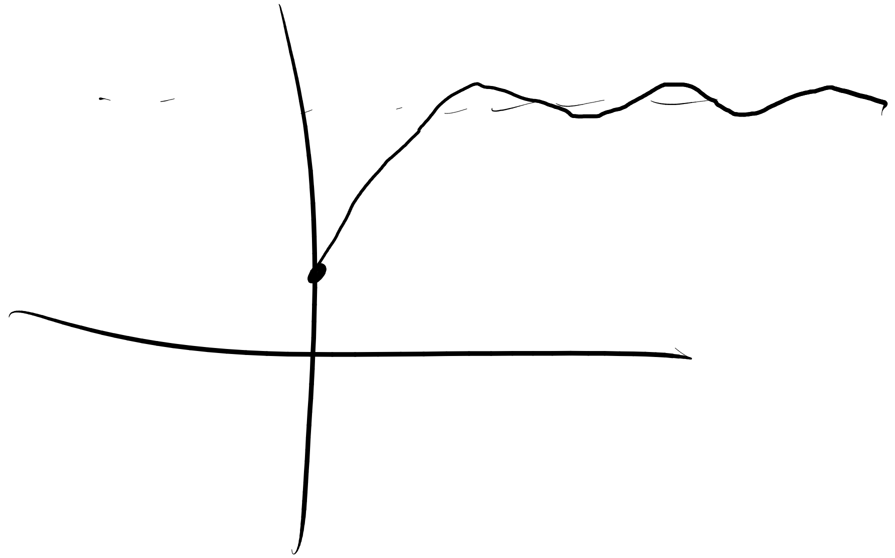
D = # buyers

S = # sellers

$$p'(t) = k(a-r) - k(s+b)p(t)$$

$$D(p) = a - bp$$

$$S(p) = r + sp$$



Hooke's Law

$m = k$

$p(t) = A\sin(t) + B\cos(t)$

$p(0) = 2$

$p(\pi/4) = \sqrt{8}$

Boundary conditions

$p(0) = 2$

$p'(0) = 1$

Initial conditions

$2 = p(0) = \cancel{A\sin(0)} + B\cancel{\cos(0)}$

$\Rightarrow B = 2$

$\sqrt{8} = p(\pi/4) = A\underbrace{\sin(\pi/4)}_{\frac{\sqrt{2}}{2}} + 2\underbrace{\cos(\pi/4)}_{\frac{\sqrt{2}}{2}}$

$\Rightarrow 4 = A + 2$

$\Rightarrow A = 2.$

§3.4 Separable differential eqns

$$\frac{dy}{dx} = g(x) f(y)$$

$$\int \frac{dy}{f(y)} = \int g(x)\, dx$$

$$\frac{dy}{dx} = ky$$

$$\int \frac{dy}{y} = \int k\, dx$$

$$\ln|y| = kx + C$$

$$y = e^{kx+C} = e^{kx} \cdot e^{C}$$

$$= P e^{kx}.$$

$$\ln|y| + C_1 = kx + C_2$$

$$\ln|y| = kx + C_3$$

$$\frac{dy}{dx} = \frac{x}{y}$$

Initial conditions $y(0) = 2$

$$\int y\,dy = \int x\,dx$$

$$\frac{y^2}{2} = \frac{x^2}{2} + C$$

$$y' = \frac{1}{\cancel{2}\sqrt{x^2+4}} \cdot \cancel{2}x = \frac{x}{y}.$$

$$y^2 = x^2 + 2C$$

$$y = \sqrt{x^2 + K}$$

$$y = \sqrt{x^2+4}.$$

$$2 = \sqrt{0+K}$$

$$\Rightarrow K = 4$$