Math 1232 Sprin 2021
Single-Variable Calculus II Mastery Quiz 1
Due Friday, January 22

This week's mastery quiz has two topics. Please do your best on those two topics. Don't worry if you make a minor error, but try to demonstrate your mastery of the underlying material. You shouldn't spend more than 10-20 minutes on this quiz.

Feel free to consult your notes, but please don't talk about the actual quiz questions with other students in the course.

Remember that you are trying to demonstrate that you understand the concepts involved. For all these problems, justify your answers and explain how you reached them. Do not just write "yes" or "no" or give a single number.

Please upload your work as *one PDF file*. You can produce the file on your computer/tablet/whatever, or you can handwrite it and then scan it. If you have a smartphone, there are many apps that can help you produce a clean single pdf; I personally have used GeniusScan but there are many options.

2. **Topic 2: Exponents and Logarithms**

(a) Showing your work, compute $\log_3(90) + \log_3(3/2) - \log_3(5)$.
Solution:

$$\log_3(90) + \log_3(3/2) - \log_3(5) = \log_3(90 \cdot 3/2/5) = \log_3(27) = 3.$$

(b) Give an exact solution for the equation $\ln(2x + 5) = 3$.
Solution:

$$\begin{aligned} \ln(2x+5) &= 3 \\ 2x+5 &= e^3 \\ 2x &= e^3 - 5 \\ x &= \frac{e^3-5}{2}. \end{aligned}$$

(c) Compute $2^{3\log_2(5)+2\log_2(7)}$.
Solution:

$$\begin{aligned} 2^{3\log_2(5)+2\log_2(7)} &= 2^{\log_2(5^3)+\log_2(7^2)} \\ &= 2^{\log_2(5^3 7^2)} = 5^3 7^2. \end{aligned}$$

Alternatively, we can argue

$$\begin{aligned} 2^{3\log_2(5)+2\log_2(7)} &= 2^{\log_2(5^3)+\log_2(7^2)} \\ &= 2^{\log_2(5^3)} 2^{\log_2(7^2)} = 5^3 7^2. \end{aligned}$$

(d) Give an exact solution for the equation $6^{x-5} = 4$.
Solution:

$$\begin{aligned} 6^{x-5} &= 4 \\ x-5 &= \log_6(4) \\ x &= 5 + \log_6(4). \end{aligned}$$

1. **Topic 1: Inverse Functions**

(a) Is $f(x) = x^2 + x$ invertible or not? Justify your answer.
Solution: We have $f(-1) = f(0) = 0$ so thsi function is not one-to-one, and thus not invertible.

(b) Find a formula for the inverse of $g(x) = 2 + \sqrt{3 - x}$.
Solution:

$$\begin{aligned} y &= 2 + \sqrt{3-x} \\ y-2 &= \sqrt{3-x} \\ (y-2)^2 &= 3 - x \\ x &= 3 - (y-2)^2 \end{aligned}$$

so $f^{-1}(y) = 3-(y-2)^2$. (You can use whichever variable you like in your formula.)

(c) Let $h(x) = \sqrt{x^3 + x + 6}$. Compute $(h^{-1})'(4)$.

Solution: By the Inverse Function Theorem, we know that

$$(h^{-1})'(4) = \frac{1}{h'(h^{-1}(4))}.$$

Guess and check shows that $h(2) = 4$ so $h^{-1}(4) = 2$. And we know that

$$h'(x) = \frac{1}{2}(x^3 + x + 6)^{-1/2}(3x^2 + 1)$$

and thus

$$h'(2) = \frac{1}{2}(16)^{-1/2}(13) = \frac{13}{8}.$$

Thus

$$(h^{-1})'(4) = \frac{1}{13/8} = \frac{8}{13}.$$