Math 1232 Sprin 2021
Single-Variable Calculus II Mastery Quiz 10
Due Friday, April 9

This week's mastery quiz has ten topics. You should do topics 20 and 19, and optionally *one* of the previous topics. Don't worry if you make a minor error, but try to demonstrate your mastery of the underlying material. You shouldn't spend more than 20-30 minutes on this quiz.

Feel free to consult your notes, but please don't talk about the actual quiz questions with other students in the course.

Remember that you are trying to demonstrate that you understand the concepts involved. For all these problems, justify your answers and explain how you reached them. Do not just write "yes" or "no" or give a single number.

Please upload your work as *one PDF file*. You can produce the file on your computer/tablet/whatever, or you can handwrite it and then scan it. If you have a smartphone, there are many apps that can help you produce a clean single pdf; I personally have used GeniusScan but there are many options.

20. Power Series as Functions
19. Power Series
18. Absolute and Conditional Convergence
17. Comparison Test and Limit Comparison Test
16. Divergence and Integral tests
15. Geometric and Telescoping Series
14. Sequences
13. Separable Differential Equations
9. Partial Fractions
7. Integration by Parts

20. **Power Series as Functions**

(a) Write a power series expression for $\frac{2x^2}{4x+1}$ centered a 0. What is the radius of convergence?

(b) If $f(x) = \sum_{n=0}^{\infty} \frac{n+1}{n!+1} x^n$, compute $\int_3^5 f(x)$.

19. **Power Series**

(a) Find the radius of convergence and the interval of convergence of $\sum_{n=0}^{\infty} \frac{(2x-5)^n}{n^2}$.

(b) Find the radius of convergence and the interval of convergence of $\sum_{n=0}^{\infty} \frac{n^2 x^n}{1 \cdot 3 \cdot 5 \cdot \cdots \cdot (2n-1)}$.

18. **Absolute and Conditional Convergence**

For each series, tell whether it absolutely converges, conditionally converges, or diverges. Justify your answer (and in particular, if it conditionally converges, explain why it doesn't absolutely converge).

(a) $\displaystyle\sum_{n=1}^{\infty} \frac{(-2)^n}{n^3+n}$

(b) $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n(3n^2+5n+2)^n}{(5n^2-3)^n}$

(c) $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n\sqrt{n}}{2n+3}$

17. **Comparison Tests**

Determine whether each of the following series converges by using an appropriate comparison test.

(a) $\displaystyle\sum_{n=1}^{\infty} \frac{n^3 + n - 1}{n^5 - 3n^4}$

(b) $\displaystyle\sum_{n=1}^{\infty} \frac{\ln(n) + n}{n^2 - 1}$

16. **Divergence and Integral Tests**

Determine whether each of the following series converges or diverges. Justify your answers using only the divergence and integral tests (and *not* the comparison tests or ratio test or root test).

(a) $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n^2+4}$

(b) $\displaystyle\sum_{n=1}^{\infty} \frac{3n^2+5}{5n^2-3n+2}$

(c) $\displaystyle\sum_{n=1}^{\infty} \frac{n+1}{n^2+2n}$

15. **Geometric and Telescoping Series**

Compute the following infinite sums, with justification:

(a) $\displaystyle\sum_{n=1}^{\infty} \frac{7^{n+1}}{2 \cdot 3^{2n-1}} =$

(b) $\displaystyle\sum_{n=1}^{\infty} \frac{(-3)^n}{2^n} =$

(c) $\displaystyle\sum_{n=1}^{\infty} \ln\left(\frac{n+4}{n+3}\right) =$

14. **Sequences**

(a) Consider the sequence $(a_n) = (3, {}^{6}\!/_{2}, {}^{9}\!/_{6}, {}^{12}\!/_{24}, {}^{15}\!/_{120}, \dots)$. Find a formula for the nth term a_n. Compute $\lim_{n\to\infty} a_n$.

(b) Let $b_n = \tan\left(\frac{(2n-1)\pi}{4}\right)$. Compute the first four terms of the sequence, and compute $\lim_{n\to\infty} b_n$, with justification.

(c) Let $c_n = \frac{5n+1}{3n-2}$. Compute the first four terms of this sequence, and compute $\lim_{n\to\infty} c_n$, with justification.

13. **Differential Equations**

(a) Find a general solution to the equation $y' = x^2/y^3$.

(b) Find a (specific) solution to the initial value problem $y' = xy - x$ if $y(0) = e + 1$

9. **Partial Fractions**

(a) Compute $\int \frac{x^2+x}{x-1}\,dx =$

(b) Compute $\int \frac{4+x}{(1+2x)(3-x)}\,dx =$

7. **Integration by Parts**

Use integration by parts to compute:

(a) $\int e^{-t}\cos(3t)\,dt =$

(b) $\int (x-3)\sin(\pi x)\,dx =$