

Math 1232 Spring 2021
Single-Variable Calculus II Mastery Quiz 11
Due Friday, April 16

This week's mastery quiz has nine topics. You should do topics 22 and 21, and optionally *one* of the previous topics. Don't worry if you make a minor error, but try to demonstrate your mastery of the underlying material. You shouldn't spend more than 20-30 minutes on this quiz.

Feel free to consult your notes, but please don't talk about the actual quiz questions with other students in the course.

Remember that you are trying to demonstrate that you understand the concepts involved. For all these problems, justify your answers and explain how you reached them. Do not just write "yes" or "no" or give a single number.

Please upload your work as *one PDF file*. You can produce the file on your computer/tablet/whatever, or you can handwrite it and then scan it. If you have a smartphone, there are many apps that can help you produce a clean single pdf; I personally have used GeniusScan but there are many options.

22. Computing Taylor Series
21. Theory of Taylor Series
20. Power Series as Functions
19. Power Series
18. Absolute and Conditional Convergence
17. Comparison Test and Limit Comparison Test
16. Divergence and Integral tests
3. Derivatives of Exponentials and Logs
2. Exponents and Logarithms

22. Computing Taylor Series

(a) Using series we already know, write down a formula for the (infinite) Taylor series for x^3e^{2x} , and then write down the degree-six polynomial explicitly.

(b) Using series we already know, write down a formula for the (infinite) Taylor series for $(1 - 2x)^{-3}$, and then write down the degree-four polynomial explicitly.

21. Theory of Taylor Series

(a) Let $f(x) = \cos^2(x)$. Use *the definition of a Taylor series* to find $T_4(x, \pi)$ for this function. (That is, find the terms up through the cubic term.)

(b) Using the Taylor series remainder, show that $\sin(x)$ is equal to its Maclaurin series.

20. Power Series as Functions

- (a) Write a power series expression for $\frac{x^4}{2-4x}$ centered at 0. What is the radius of convergence?

- (b) If $f(x) = \sum_{n=0}^{\infty} 2^n n^3 (x-2)^n$, compute $\frac{d}{dx} f(x)$ and $\int f(x) dx$.

19. Power Series

(a) Find the radius of convergence and the interval of convergence of $\sum_{n=0}^{\infty} \frac{(5x-3)^n}{\sqrt{n}}$.

(b) Find the radius of convergence and the interval of convergence of $\sum_{n=0}^{\infty} \frac{n}{5^n} (x-3)^n$.

18. Absolute and Conditional Convergence

For each series, tell whether it absolutely converges, conditionally converges, or diverges. Justify your answer (and in particular, if it conditionally converges, explain why it doesn't absolutely converge).

$$(a) \sum_{n=1}^{\infty} \frac{(-1)^n}{n^2}$$

$$(b) \sum_{n=1}^{\infty} \frac{(-1)^n}{3n+2}$$

$$(c) \sum_{n=1}^{\infty} \frac{(-1)^n}{3^n+2}$$

17. Comparison Tests

Determine whether each of the following series converges by using an appropriate comparison test.

$$(a) \sum_{n=1}^{\infty} \frac{n^2 + e^{-n}}{n^3 - \ln(n^3 + 1)}$$

$$(b) \sum_{n=1}^{\infty} \frac{n^2 + \sqrt{n} + 2}{n^4 - n^{4/3} + 2}$$

16. Divergence and Integral Tests

Determine whether each of the following series converges or diverges. Justify your answers using only the divergence and integral tests (and *not* the comparison tests or ratio test or root test).

$$(a) \sum_{n=1}^{\infty} \frac{n \ln(n)}{\ln(\ln(\ln(n)))}$$

$$(b) \sum_{n=1}^{\infty} \frac{n^3}{n^4 + 7}$$

$$(c) \sum_{n=1}^{\infty} n e^{-n^2+1}$$

3. Derivatives of Exponentials and Logarithms

(a) Compute $\frac{d}{dx} x^{\sqrt{x^2+1}}$

(b) Find an equation for the tangent line to the curve $y = \frac{e^x}{x^2}$ at the point $(1, e)$.

(c) Compute $\frac{d}{dx} \cos(\ln |x - 3|)$.

2. Topic 2: Exponents and Logarithms

(a) Showing your work, compute $\log_3(15) + 2\log_3(6) - \log_3(20)$. (Give an exact answer with no decimals.)

(b) Give an exact solution for the equation $e^{3x^2-2} = 4$.

(c) Compute $4^{\log_2(11) - 3\log_2(3)}$. (Give an exact answer with no decimals.)

(d) Give an exact solution for the equation $\log_5(7x - 24) = 2$.