

Math 1232 Spring 2021
Single-Variable Calculus II Mastery Quiz 12
Due Friday, April 23

This week's mastery quiz has fourteen topics. You should do topics 24 and 23, and optionally *one* of the previous topics. Don't worry if you make a minor error, but try to demonstrate your mastery of the underlying material. You shouldn't spend more than 20-30 minutes on this quiz.

Feel free to consult your notes, but please don't talk about the actual quiz questions with other students in the course.

Remember that you are trying to demonstrate that you understand the concepts involved. For all these problems, justify your answers and explain how you reached them. Do not just write "yes" or "no" or give a single number.

Please upload your work as *one PDF file*. You can produce the file on your computer/tablet/whatever, or you can handwrite it and then scan it. If you have a smartphone, there are many apps that can help you produce a clean single pdf; I personally have used GeniusScan but there are many options.

24. Parametrization
23. Applications of Taylor Series
22. Computing Taylor Series
21. Theory of Taylor Series
20. Power Series as Functions
19. Power Series
18. Absolute and Conditional Convergence
17. Comparison Test and Limit Comparison Test
15. Geometric and Telescoping Series
13. Separable Differential Equations
11. Improper Integrals
10. Numeric Integration
6. L'Hospital's Rule
4. Integrals involving Exponentials and Logs

24. Parametrization

(a) Find a parametrization of the ellipse $x^2 + y^2/4 = 1$. (Hint: what are the x and y intercepts?)

(b) Find an equation of the line tangent to the curve $x = 1 + \sqrt{t}, y = t^3$ at the point $(2, 1)$.

(c) Find the length of the curve parametrized by $x = e^t - t, y = 4e^{t/2}$ for $0 \leq t \leq 2$.

23. Applications of Taylor Series

(a) Use a Taylor series to compute $\lim_{x \rightarrow 0} \frac{\cos(x^2) - 1 + x^4/2}{x^8} =$

(b) Use a degree-three Taylor polynomial to estimate $\sqrt{1.2}$.

(c) Use a degree-five Taylor polynomial to estimate $\arctan(.1)$.

22. Computing Taylor Series

(a) Using series we already know, write down a formula for the (infinite) Taylor series for $x^2 \ln(1 - 2x^3)$, and then write down the degree-eleven polynomial explicitly.

(b) Using series we already know, write down a formula for the (infinite) Taylor series for $(1 + 3x)^{2/3}$, and then write down the degree-three polynomial explicitly.

21. Theory of Taylor Series

- (a) Let $f(x) = e^{x^2}$. Use *the definition of a Taylor series* to find $T_4(x, 0)$ for this function. (That is, find the terms up through the degree-four term.)

- (b) Estimate the error you use if $T_3(x) = x - \frac{x^2}{2} + \frac{x^3}{3}$ to approximate $g(x) = \ln(1 + x)$ at $x = .5$.

20. Power Series as Functions

- (a) Write a power series expression for $\frac{2x^3}{5-x}$ centered at 0. What is the radius of convergence?

- (b) If $f(x) = \sum_{n=0}^{\infty} \frac{3^n}{n!} (x+2)^n$, compute $\frac{d}{dx} f(x)$ and $\int f(x) dx$.

19. Power Series

(a) Find the radius of convergence and the interval of convergence of $\sum_{n=0}^{\infty} (n(x-3))^n$.

(b) Find the radius of convergence and the interval of convergence of $\sum_{n=0}^{\infty} \frac{2^n}{n^2+n} x^n$.

18. Absolute and Conditional Convergence

For each series, tell whether it absolutely converges, conditionally converges, or diverges. Justify your answer (and in particular, if it conditionally converges, explain why it doesn't absolutely converge).

$$(a) \sum_{n=1}^{\infty} \frac{n}{3^n}$$

$$(b) \sum_{n=1}^{\infty} \frac{5^n}{5^n + 1}$$

$$(c) \sum_{n=1}^{\infty} \frac{(-2)^n}{n2^n + 1}$$

17. Comparison Tests

Determine whether each of the following series converges by using an appropriate comparison test.

$$(a) \sum_{n=1}^{\infty} \frac{3n^3 - 2n^2 + n}{4n^6 + 4n^3 - n}$$

$$(b) \sum_{n=1}^{\infty} \frac{n^3}{n^4 \cos^2(n^2) - n}$$

15. Geometric and Telescoping Series

Compute the following infinite sums, with justification:

$$(a) \sum_{n=1}^{\infty} \frac{5^{n-3}}{2^n} =$$

$$(b) \sum_{n=1}^{\infty} \frac{(-4)^{n-1}}{5^n} =$$

$$(c) \sum_{n=1}^{\infty} \frac{-1}{n^2 + 5n + 6} =$$

13. Differential Equations

(a) Find a general solution to the equation $y' = e^{x+y}$.

(b) Find a (specific) solution to the initial value problem $x + y\sqrt{x^2 + 1}y' = 0$ with $y(0) = 2$.

11. Improper Integrals

(a) Compute $\int_0^7 \frac{1}{\sqrt[3]{7-x}} dx$.

(b) Compute $\int_{-\infty}^{\infty} x e^{-x^2} dx$.

10. Numeric Integration

- (a) How many intervals do you need with the **trapezoid** rule to approximate $\int_2^5 \frac{x}{x+1}$ to within $1/200$? Compute that approximation. (Feel free to use a calculator to plug in numeric values, or to leave the answer in exact unsimplified terms, but show every step.)

- (b) Suppose we have

$$g(0) = 5 \quad g(1) = 4 \quad g(2) = 2 \quad g(3) = 3 \quad g(4) = 5 \quad g(5) = 6 \quad g(6) = 5$$

Approximate $\int_0^6 g(x) dx$ using the midpoint rule and the Simpson's rule.

6. L'Hospital's Rule

Compute the following limits:

$$(a) \lim_{x \rightarrow 1} \frac{x^7 - 1}{x^4 - 1} =$$

$$(b) \lim_{x \rightarrow \infty} \sqrt{x} e^{-x} =$$

$$(c) \lim_{x \rightarrow 0} (1 - 3x)^{1/x} =$$

4. Integrals Involving Exponentials and Logarithms

Compute the following integrals:

$$(a) \int_1^2 \frac{e^{1/x}}{x^2} dx =$$

$$(b) \int \frac{1 + 2x + x^2}{3x + 3x^2 + x^3} dx =$$

$$(c) \int \frac{2^x}{2^x + 3} dx =$$