Math 1232 Spring 2021
Single-Variable Calculus II Mastery Quiz 13
Due Friday, April 30

This week's mastery quiz has thirteen topics. You can submit up to three topics. Don't worry if you make a minor error, but try to demonstrate your mastery of the underlying material. You shouldn't spend more than 20-30 minutes on this quiz.

Feel free to consult your notes, but please don't talk about the actual quiz questions with other students in the course.

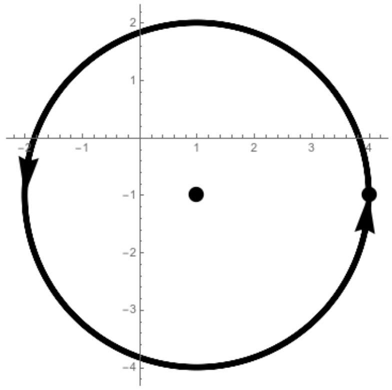
Remember that you are trying to demonstrate that you understand the concepts involved. For all these problems, justify your answers and explain how you reached them. Do not just write "yes" or "no" or give a single number.

Please upload your work as *one PDF file*. You can produce the file on your computer/tablet/whatever, or you can handwrite it and then scan it. If you have a smartphone, there are many apps that can help you produce a clean single pdf; I personally have used GeniusScan but there are many options.

24. Parametrization
23. Applications of Taylor Series
22. Computing Taylor Series
21. Theory of Taylor Series
20. Power Series as Functions
19. Power Series
18. Absolute and Conditional Convergence
17. Comparison Test and Limit Comparison Test
15. Geometric and Telescoping Series
14. Sequences
12. Arc Length and Surface Area
11. Improper Integrals
10. Numeric Integration
4. Integrals involving exponents and logs
1. Inverse Functions

24. **Parametrization**

(a) Find a parametrization for the circle of radius 3 centered at $(1, -1)$, starting at $(4, -1)$ and going counterclockwise twice around the circle.



(b) Find an equation of the line tangent to the curve $x = \cos^3(t), y = \sin^3(t)$ at the point $(1/8, -3\sqrt{3}/8)$.

(c) Find the length of the curve parametrized by $x = 3t^2, y = 3t - t^3$, for $1 \le t \le 4$.

23. **Applications of Taylor Series**

(a) Use a Taylor series to compute $\lim_{x \to 0} \frac{xe^{x^3} - x - x^4}{x^7} =$

(b) Use a degree-three Taylor polynomial to estimate $(1.1)^{3.1}$.

(c) Use a degree-five Taylor polynomial to estimate $\sin(.3)$.

22. **Computing Taylor Series**

(a) Using series we already know, write down a formula for the (infinite) Taylor series for $x(8+x)^{5/3}$, and then write down the degree-four polynomial explicitly.

(b) Using series we already know, write down a formula for the (infinite) Taylor series for $e^{3x} - e^{x}$, and then write down the degree-three polynomial explicitly.

21. **Theory of Taylor Series**

(a) Let $f(x) = \sin(x)$. Use *the definition of a Taylor series* to find $T_3(x, \pi/6)$ (centered at $\pi/6$) for this function. (That is, find the terms up through the degree-three term.)

(b) Estimate the error if you use $T_4(x) = 1 - \frac{x^2}{2} + \frac{x^4}{4!}$ to approximate $g(x) = \cos(x)$ at $x = -.5$.

20. **Power Series as Functions**

(a) Write a power series expression for $\frac{x}{2+x^2}$ centered a 0. What is the radius of convergence?

(b) If $f(x) = \sum_{n=1}^{\infty} \frac{2^n}{n^2+n}(x-5)^n$, compute $\frac{d}{dx} f(x)$ and $\int f(x)\,dx$.

19. **Power Series**

(a) Find the radius of convergence and the interval of convergence of $\sum_{n=0}^{\infty} \frac{(n!)^2}{(3n)!}(x+2)^n$.

(b) Find the radius of convergence and the interval of convergence of $\sum_{n=1}^{\infty} \frac{3^n(x+2)^n}{\ln(3n)}$.

18. **Absolute and Conditional Convergence**

For each series, tell whether it absolutely converges, conditionally converges, or diverges. Justify your answer (and in particular, if it conditionally converges, explain why it doesn't absolutely converge).

(a) $\displaystyle\sum_{n=2}^{\infty} \frac{(-n)^3}{n^4 - 4}$

(b) $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n (3n+1)^n}{(5n^2 - 2)^n}$

(c) $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n}{n + 1.1^n}$

17. **Comparison Tests**

Determine whether each of the following series converges by using an appropriate comparison test.

(a) $\displaystyle\sum_{n=1}^{\infty} \frac{\sin(1/n)}{n^2}$

(b) $\displaystyle\sum_{n=1}^{\infty} \frac{n}{n^3 - n\sin(n)}$

15. **Geometric and Telescoping Series**

Compute the following infinite sums, with justification:

(a) $\displaystyle\sum_{n=1}^{\infty} \frac{5^{n+1}}{3^{2n+1}} =$

(b) $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n^2 + 11n + 30} =$

(c) $\displaystyle\sum_{n=1}^{\infty} \frac{3^{3n-2}}{7^{n+5}} =$

14. **Sequences**

(a) Consider the sequence $(a_n) = (2, {}^3\!/\!{}_4, {}^4\!/\!{}_9, {}^5\!/\!{}_{16}, {}^6\!/\!{}_{25}, \ldots)$. Find a formula for the nth term a_n. Compute $\lim_{n\to\infty} a_n$.

(b) Let $b_n = \frac{n!+2}{(n+2)!}$. Compute the first four terms of the sequence, and compute $\lim_{n\to\infty} b_n$, with justification.

(c) Let $c_n = \frac{\sqrt{n}}{\sqrt{n+2}}$. Compute the first four terms of this sequence, and compute $\lim_{n\to\infty} c_n$, with justification.

12. **Arc Length and Surface Area**

(a) Compute the arc length of the curve $y = \frac{1}{27}(9x^2 + 6)^{3/2}$ as x varies from 2 to 4.

(b) Compute the area of the surface obtained by taking the curve $y = x^3$ as x goes from 0 to 1 and rotating it around the x-axis.

11. **Improper Integrals**

(a) Compute $\displaystyle\int_1^\infty \frac{\ln(x)}{x}\,dx$.

(b) Compute $\displaystyle\int_{-1}^1 \frac{1}{\sqrt[3]{x^2}}\,dx$.

10. **Numeric Integration**

(a) How many intervals do you need with the **trapezoid** rule to approximate $\int_0^3 \frac{1}{1+x}$ to within 1/2? Compute that approximation. (Feel free to use a calculator to plug in numeric values, or to leave the answer in exact unsimplified terms, but show every step.)

(b) Suppose we have

$$g(0) = 2 \quad g(.5) = 3 \quad g(1) = 4 \quad g(1.5) = 5 \quad g(2) = 3 \quad g(2.5) = 2 \quad g(3) = 1$$

Approximate $\int_0^3 g(x)\,dx$ using the midpoint rule and Simpson's rule.

4. **Integrals Involving Exponentials and Logarithms**

Compute the following integrals:

(a) $\int \sec^2(x) e^{\tan(x)}\, dx =$

(b) $\int \frac{1}{x \ln(x) \ln(\ln(x))}\, dx =$

(c) $\int_0^2 x 2^{x^2}\, dx$

1. **Topic 1: Inverse Functions**

   (a) Is $f(x) = \sqrt{x^4 + 3}$ invertible or not? Justify your answer.

   (b) Find a formula for the inverse of $g(x) = \ln(x^3 - 2)$.

   (c) Let $h(x) = \sqrt{5x^3 + 3x + 1}$. Compute $(h^{-1})'(3)$.