

Math 1232 Spring 2021
Single-Variable Calculus II Mastery Quiz 5
Due Friday, February 19

This week's mastery quiz has ten topics. You should do topics 10 and 9, and optionally one of the previous topics. Don't worry if you make a minor error, but try to demonstrate your mastery of the underlying material. You shouldn't spend more than 20-30 minutes on this quiz.

Feel free to consult your notes, but please don't talk about the actual quiz questions with other students in the course.

Remember that you are trying to demonstrate that you understand the concepts involved. For all these problems, justify your answers and explain how you reached them. Do not just write "yes" or "no" or give a single number.

Please upload your work as *one PDF file*. You can produce the file on your computer/tablet/whatever, or you can handwrite it and then scan it. If you have a smartphone, there are many apps that can help you produce a clean single pdf; I personally have used GeniusScan but there are many options.

10. Numeric Integration
9. Partial Fractions
8. Trigonometric integrals
7. Integration by parts
6. L'Hospital's Rule
5. Inverse Trigonometric Functions
4. Integrals involving Exponentials and Logarithms
3. Derivatives of Exponentials and Logarithms
2. The Exponential and the Logarithm
1. Inverse Functions

10. Numeric Integration

- (a) Let $f(x) = x^3 + x$. How many intervals do you need with the midpoint rule to approximate $\int_1^2 x^3 + x \, dx$ to within $1/10$? Compute that approximation. (Feel free to use a calculator to plug values into f , but show every step.)

- (b) Suppose we have

$$g(0) = 2.4 \quad g(1) = 3 \quad g(2) = 2.7 \quad g(3) = 2.3 \quad g(4) = 1.9$$

Approximate $\int_0^4 g(x) \, dx$ using the Trapezoid rule and using Simpson's rule.

9. Partial Fractions

(a) Compute $\int \frac{x^3}{x-1} dx =$

(b) Compute $\int \frac{3x}{(x+4)(x-2)} dx =$

8. Trigonometric Integrals

Compute

(a) $\int \sec^4(3t) dt =$

(b) $\int x^3 \sqrt{1-x^2} dx =$

7. Integration by Parts

Compute:

(a) $\int e^x \sin(5x) dx =$

(b) $\int_{\pi/9}^{\pi/6} x \cos(3x) dx =$

6. L'Hospital's Rule

Compute the following limits:

$$(a) \lim_{x \rightarrow -\infty} \frac{e^x}{\arctan(x) + \pi/2} =$$

$$(b) \lim_{x \rightarrow 3} \frac{x^2 - x - 6}{\sin(x - 3)} =$$

$$(c) \lim_{x \rightarrow 0} x^x =$$

5. Inverse Trigonometric Functions

(a) Compute $\arccos\left(\frac{\sqrt{2}}{2}\right) =$

(b) Compute $\tan(\arcsin(2/5))$.

(c) $\frac{d}{dx} \frac{1}{\arcsin(x^2)} =$

(d) $\int \frac{1}{x\sqrt{4 - \ln(x)^2}} dx =$

4. Integrals Involving Exponentials and Logarithms

Compute the following integrals:

$$(a) \int_1^2 \frac{e^{1/x}}{x^2} dx =$$

$$(b) \int x^5 + 5^x dx =$$

$$(c) \int \frac{\cos(x) \sin(x)}{1 + \cos^2(x)} dx =$$

3. Derivatives of Exponentials and Logarithms

(a) Compute $\frac{d}{dx} \sin(x)^{\tan(x)}$.

(b) Compute $\frac{d}{dx} \pi^{\sqrt{x}}$

(c) Find an equation for the tangent line to the curve $y = \log_2(x^3 + x)$ at the point $(1, 1)$.

2. Exponents and Logarithms

(a) Showing your work, compute $2 \log_9(6) + \log_9(15) - \log_9(20)$. (Give an exact answer with no decimals.)

(b) Compute $4^{2 \log_2(3) - 4 \log_2(5)}$. (Give an exact answer with no decimals.)

(c) Give an exact solution for the equation $5^{x^2+3x+4} = 25$.

(d) Give an exact solution for the equation $\ln(4x + 2) = 7$.

1. Topic 1: Inverse Functions

(a) Is $f(x) = e^{x^3+1}$ invertible or not? Justify your answer. If it's invertible, give a formula for the inverse.

(b) Is $g(x) = x^4 + x$ invertible or not? Justify your answer. If it's invertible, give a formula for the inverse.

(c) Let $h(x) = e^{x^3+x}$. Compute $(h^{-1})'(e^2)$.