Math 1232 Sprin 2021
Single-Variable Calculus II Mastery Quiz 7
Due Friday, March 12

This week's mastery quiz has eleven topics. You should do topics 16 and 15, and optionally *one* of the previous topics. Don't worry if you make a minor error, but try to demonstrate your mastery of the underlying material. You shouldn't spend more than 20-30 minutes on this quiz.

Feel free to consult your notes, but please don't talk about the actual quiz questions with other students in the course.

Remember that you are trying to demonstrate that you understand the concepts involved. For all these problems, justify your answers and explain how you reached them. Do not just write "yes" or "no" or give a single number.

Please upload your work as *one PDF file*. You can produce the file on your computer/tablet/whatever, or you can handwrite it and then scan it. If you have a smartphone, there are many apps that can help you produce a clean single pdf; I personally have used GeniusScan but there are many options.

18. Absolute and Conditional Convergence

17. Comparison Test and Limit Comparison Test

16. Divergence and Integral tests

15. Geometric and Telescoping Series

14. Sequences

13. Differential Equations

12. Arc Length and Surface Area

11. Improper Integrals

10. Numeric Integration

6. L'Hospital's Rule

5. Inverse Trigonometric Functions

18. **Absolute and Conditional Convergence**

For each series, tell whether it absolutely converges, conditionally converges, or diverges. Justify your answer (and in particular, if it conditionally converges, explain why it doesn't absolutely converge).

(a) $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n n}{n^2+1}$

(b) $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n 3^n}{5^n+1}$

(c) $\displaystyle\sum_{n=1}^{\infty} (-1)^n \left(\frac{2n^2+n+1}{n^2-3n+2}\right)^n$

17. **Comparison Tests**

Determine whether each of the following series converges by using an appropriate comparison test.

(a) $\displaystyle\sum_{n=1}^{\infty} \frac{n \sin^2(n)}{n^3 + 2}$

(b) $\displaystyle\sum_{n=1}^{\infty} \frac{n^2 - 3}{n^3 + 2}$

16. **Divergence and Integral Tests**

Determine whether each of the following series converges or diverges. Justify your answers using only the divergence and integral tests (and *not* the comparison tests).

(a) $\displaystyle\sum_{n=1}^{\infty} \sqrt{n^3+1}$

(b) $\displaystyle\sum_{n=1}^{\infty} ne^{-n^2}$

(c) $\displaystyle\sum_{n=1}^{\infty} \frac{\sqrt{n}}{\sqrt{n^3}+1}$

15. **Geometric and Telescoping Series**

Compute the following infinite sums, with justification:

(a) $\displaystyle\sum_{n=1}^{\infty} \frac{2^n}{3 \cdot 5^{n-1}} =$

(b) $\displaystyle\sum_{n=1}^{\infty} \frac{2}{n^2 + 4n + 3} =$

(c) $\displaystyle\sum_{n=1}^{\infty} \frac{2^{2n+1}}{3^n} =$

14. **Sequences**

(a) Consider the sequence $(a_n) = \left(\frac{1}{2}, 2/5, 3/10, 4/17, \dots\right)$. Find a formula for the nth term a_n. Compute $\lim_{n\to\infty} a_n$.

(b) Let $b_n = \frac{n!}{2^n}$. Compute the first four terms of the sequence, and compute $\lim_{n\to\infty} b_n$, with justification.

(c) Let $c_n = \frac{\log_2(n)}{n}$. Compute the first four terms of this sequence, and compute $\lim_{n\to\infty} c_n$, with justification.

13. **Differential Equations**

(a) Find a general solution to the equation $y' = xe^x y$.

(b) Find a (specific) solution to the initial value problem $y'/x = \cos^2(y)$ if $y(0) = \pi/3$

12. **Arc Length and Surface Area**

(a) Set up (but don't compute!) an integral for the arc length of the curve $y = 1 + 3x + e^x$ as y goes from 2 to $4 + e$.

(b) Compute the area of the surface obtained by taking the curve $x^{2/3} + y^{2/3} = 1$ as x goes from 0 to 1 and rotating it around the y-axis.

11. **Improper Integrals**

(a) Compute $\displaystyle\int_{-1}^{1} \frac{1}{\sqrt[3]{x^2}}\,dx.$

(b) Compute $\displaystyle\int_{1}^{\infty} xe^{-x^2}\,dx.$

10. **Numeric Integration**

(a) How many intervals do you need with the trapezoid rule to approximate $\int_0^\pi \sin(x)\,dx$ to within 1/4? Compute that approximation.

(b) Suppose we have

$$g(3) = 2 \quad g(5) = 5 \quad g(7) = 3 \quad g(9) = 7 \quad g(11) = 8 \quad g(13) = 9 \quad g(15) = 1$$

Approximate $\int_3^9 g(x)\,dx$ using the midpoint rule and Simpson's rule.

6. **L'Hospital's Rule**

Compute the following limits:

(a) $\lim_{x\to\infty} \dfrac{1+\ln(x)+(\ln(x))^2}{\sqrt{x}} =$

(b) $\lim_{s\to 0} \dfrac{7^s - 3^{2s}}{s} =$

(c) $\lim_{x\to 0} (2x+1)^{\cot(x)} =$

5. **Inverse Trigonometric Functions**

(a) Compute $\arcsin\left(-\sqrt{3}/2\right) =$

(b) Compute $\sin(\arccos(5/13))$.

(c) $\dfrac{d}{dx}\arctan(x^3 + 1/x) =$

(d) $\displaystyle\int \frac{1}{\sqrt{9-4x^2}}\,dx.$