

Math 4981 Spring 2021
Cryptography HW 7
Due Thursday, March 4

1. Let m be an integer, and let x be an integer with $\gcd(x, m) = 1$. Prove that $x^{\phi(m)-a}$ is an inverse of $x^a \pmod m$.
2. (a) What is $\phi(3^2)$? What is $\phi(3^3)$?
(b) What is $\phi(5^2)$? What is $\phi(5^3)$?
(c) Let p be a prime number. Find and prove a formula for $\phi(p^n)$.
3. Compute:
 - (a) $\text{ord}_{13} 5$
 - (b) $\text{ord}_{13} 7$
 - (c) $\text{ord}_{13} 2$
 - (d) $\text{ord}_{127} 2$
4. (a) What is the inverse of $19 \pmod{96}$?
(b) Use your answer in part (a) to solve the congruence $x^{19} \equiv 36 \pmod{97}$.
5. Suppose Alice and Bob are using the prime $p = 1373$ and the base $g = 2$ for an ElGamal cryptosystem.
 - (a) Alice chooses $a = 947$ as her private key. What is the value of her public key A ?
 - (b) Now suppose Bob chooses $b = 716$ as his private key, and thus his public key is $469 \pmod{1373}$. Alice encrypts the message $m = 583$ using the ephemeral key $k = 877$. What is the ciphertext Alice sends to Bob?
 - (c) Alice chooses a new private key $a = 299$ with associated public key $A \equiv 34 \pmod{1373}$. Bob encrypts a message and sends the ciphertext $(c_1, c_2) = (661, 1325)$. What is the message?
6. Alice publishes an RSA public key with modulus $N = 2038667$ and exponent $e = 103$.
 - (a) Bob wants to send Alice the message $m = 892383$. What ciphertext does he send her?
 - (b) Alice knows that N factors into two primes, one of which is 1301 . What is her decryption exponent d ?

- (c) Some time later, Alice receives the ciphertext $c = 317730$ from Bob. What is the message?
7. Suppose Eve knows that $N = pq = 352717$, and also intercepts the fact that $(p-1)(q-1) = 351520$. Can you determine $p+q$ from this? Can you determine p and q (without directly factoring N)?