Math 1231 Practice Final Solutions

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- These are the instructions you will see on the real test, next week. I include them here so you know what to expect.
- You will have 120 minutes for this test.
- You are not allowed to consult books or notes during the test, but you may use a one-page, two-sided, handwritten cheat sheet you have made for yourself ahead of time.
- You may not use a calculator. You may leave answers unsimplified, except you should compute trigonometric functions as far as possible.
- The exam has 13 problems, one on each mastery topic. The exam has 12 pages total.
- Each part of each major topic is worth 10 points. The question on topic S8 is worth 10 points.
- The questions on topics S1 through S6 are *optional*. Answering one correctly can earn you up to two bonus points on the test. More importantly, answering one correctly can raise your overall mastery score.
- Read the questions carefully and make sure to answer the actual question asked. Make sure to justify your answers—math is largely about clear communication and argument, so an unjustified answer is much like no answer at all.

  When in doubt, show more work and write complete sentences.
- If you need more paper to show work, I have extra at the front of the room.
- Good luck!

Name:

Recitation Section:

Problem 1 (M1). (a) Compute $\lim_{x \to 25} \frac{\sqrt{x} - 5}{x - 25}$

(b) Compute $\lim_{x \to -\infty} \frac{3x}{\sqrt{4x^2 + 3}}$

Problem 2 (M2). (a) Find $\dfrac{d}{dx}\sqrt[4]{\dfrac{x^3+\cos(x^2)}{\sin(x^3)+1}}$

(b) Find a formula for y' in terms of x and y if $x^8+x^4+y^4+y^6=1$.

Problem 3 (M3). (a) If $f(x) = \sqrt{x}+\tan(\pi x)$, use a linear approximation centered at 4 to estimate $f(4.1)$.

(b) A curve is defined by the equation $x^4 - 2x^2y^2 + y^4 = 16$. $(\sqrt{5}, 1)$. What is the equation of the tangent line to the curve at the point $(\sqrt{5}, 1)$?

Problem 4 (M4). (a) Find the absolute extrema of $f(x) = 3x^4 - 20x^3 + 24x^2 + 7$ on $[0, 5]$.

(b) Find and classify the critical points of $g(x) = \dfrac{2x-1}{x^2+2}$.

Problem 5 (M5). (a) Compute $\int \sin^4(t)\cos(t)\,dt$

(b) **By explicitly changing the bounds of the integral**, compute $\int_0^4 x^3\sqrt{9+x^2}\,dx$.

Problem 6 (M6). (a) What is the x-coordinate of the center of mass of the region bounded by $y = x^3$ and $x = 4$?

(b) A spring with natural length of 8 inches takes 6 pounds of force to stretch to 10 inches. Set up (but do not evaluate) an integral to compute the work done by stretching the spring from 12 inches to 16 inches. What units will this integral output?

Problem 7 (S8)**.** Using **only the definition of Riemann sum** and your knowledge of limits, compute the exact area under the curve $x^2 + x^3$ between $x = 1$ and $x = 3$.

These questions are optional. Only attempt these if you have finished all the major topics and topic S8, and want to improve your mastery scores.

Problem 8 (S1)**.** Suppose $f(x) = x^2 + 3$, and we want an output of approximately 19. If we want our input to be positive, what input a should we aim for? Find a δ so that if our input is $a \pm \delta$ then our output will be 19 ± 1. Explain how you found this δ and why it should give us what we want.

Problem 9 (S2)**.** Use the Squeeze Theorem to show that $\lim_{x \to 5}(x-5)\sin\left(\frac{x^2+1}{x-5}\right) = 0$.

These questions are optional. Only attempt these if you have finished all the major topics and topic S8, and want to improve your mastery scores.

Problem 10 (S3)**. Directly from the definition**, compute $f'(1)$ where $f(x) = \sqrt{x+3}$.

Problem 11 (S4)**.** Suppose that if a car travels at v miles per hour then its fuel efficiency is $F(v) = 8 + 1.3v - .015v^2$ miles per gallon.

(i) What does the derivative $F'(v)$ represent, and what are its units?

(ii) Compute $F'(60)$. What does this tell you?

These questions are optional. Only attempt these if you have finished all the major topics and topic S8, and want to improve your mastery scores.

Problem 12 (S5)**.** A cone with height h and base radius r has volume $\frac{1}{3}\pi r^2 h$. Suppose we have an inverted conical water tank, of height 4m and radius 6m. Water is leaking out of a small hole at the bottom of the tank. If the current water level is 2m and the water level is dropping at $\frac{1}{9\pi}$ meters per minute, what volume of water leaks out every minute?

These questions are optional. Only attempt these if you have finished all the major topics and topic S8, and want to improve your mastery scores.

Problem 13 (S6)**.** Let $j(x) = x^4 - 14x^2 + 24x + 6$. We can compute $j'(x) = 4(x+3)(x-1)(x-2)$ and $j''(x) = 4(3x^2 - 7)$. Sketch a graph of j.

Your answer should discuss the domain, asymptotes, limits at infinity, critical points and values, intervals of increase and decrease, and concavity.