Math 1231 Practice Midterm 2 Solutions

Instructor: Jay Daigle

(a) These are the instructions you will see on the real test, next week. I include them here so you know what to expect.

(b) You will have 75 minutes for this test.

(c) You are not allowed to consult books or notes during the test, but you may use a one-page, one-sided, handwritten cheat sheet you have made for yourself ahead of time.

(d) You may not use a calculator. You may leave answers unsimplified, except you should compute trigonometric functions as far as possible.

(e) The exam has 5 problems, one on each mastery topic since the first midterm. The exam has 5 pages total.

(f) Read the questions carefully and make sure to answer the actual question asked. Make sure to justify your answers—math is largely about clear communication and argument, so an unjustified answer is much like no answer at all.

When in doubt, show more work and write complete sentences.

(g) If you need more paper to show work, I have extra at the front of the room.

(h) Good luck!

Name:

Recitation Section:

Problem 1 (M3)**.**

(a) Find a tangent line to the curve given by $x^4 - 2x^2y^2 + y^4 = 16$ at the point $(\sqrt{5}, 1)$.

(b) Give equation for the linear approximation of the function $f(x) = x\sin(x)$ near the point $a = \pi/2$. Use it to estimate $f(1.5)$.

Problem 2 (M4)**.**

(a) Find and classify all the critical points of $f(x) = (x-5)\sqrt[3]{x^2}$.

(b) Find the absolute extrema of $g(x) = (x^2 - 3x)\sqrt[3]{x-3}$ on $[1, 4]$, and justify your claim that these are the absolute extrema.

Problem 3 (S4)**.** Suppose that $Q(p) = 3p^2 + 10p - 100$ is the number of widgets you can buy at a price of p dollars.

(i) What are the units of $Q'(p)$? What does it represent physically? What does it mean if $Q'(p)$ is big?

(ii) Calculate $Q'(10)$. What does this tell you physically? What physical observation could you make to check your calculation?

Problem 4 (S5)**.** The surface area of a cube is given by the formula $A = 6s^2$ where s is the length of a side. If the side lengths are increasing by 2 inches per second, how fast is the surface area increasing when the area is 54 square inches?

Problem 5 (S6)**.** Let $f(x) = \dfrac{x^3 - 2}{x^4}$. We compute that $f'(x) = \dfrac{8 - x^3}{x^5}$ and $f''(x) = \dfrac{2x^3 - 40}{x^6}$. Sketch a graph of f.

Your answer should discuss the domain, asymptotes, roots, limits at infinity, critical points and values, intervals of increase and decrease, and concavity.