Math 1231: Single-Variable Calculus 1
George Washington University Fall 2022
Recitation 2

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Problem 1. Let $f(x) = 5x + 2$. We want to use an $\varepsilon - \delta$ argument to compute $\lim_{x\to 2} f(x)$.

(a) If x is about 2, what should $f(x)$ be?

(b) Write down expressions using absolute value for the input and output errors.

(c) If we want $\varepsilon = 1$, what does δ need to be?

(d) Find a formula for δ in terms of ε (same form as $\delta = \varepsilon/3$ or $\delta = \varepsilon$).

(e) Try to write a full proof.

Problem 2 (Optional)**.** Let $g(x) = x^2$. We want to use an $\varepsilon - \delta$ argument to compute $\lim_{x\to 0} g(x)$.

(a) If x is about 0, what should $g(x)$ be?

(b) Write down expressions using absolute value for the input and output errors.

(c) If we want $\varepsilon = 1$, what does δ need to be? What about $\varepsilon = 1/4$?

(d) Find a formula for δ in terms of ε (same form as $\delta = \varepsilon/3$ or $\delta = \varepsilon$).

(e) Try to write a full proof.

Now let's look at easier ways to actually compute limits.

Problem 3. Let $f(x) = \frac{x-1}{x^2-1}$.

(a) What is $f(2)$? Is f continuous at 2?

(b) What is $\lim_{x\to 2} f(x)$?

(c) What is $f(1)$? Is f continuous at 1?

(d) What function can we find that's almost the same as f, but defined and continuous at 1? (Is this function the same as f?)

(e) What is $\lim_{x\to 1} f(x)$?

Problem 4 (Optional)**.** Let $g(x) = \frac{(x+1)^2-1}{x+2}$.

(a) Is g continuous where it's defined? Where is it undefined?

(b) Can you find a function that's almost identical to g but continuous everywhere?

(c) What is $\lim_{x\to -2} g(x)$?

Problem 5. Let $h(x) = \frac{x-1}{\sqrt{5-x}-2}$.

(a) Is this function continuous where it's defined? Where is it undefined?

(b) We can factor an $x-1$ out of the top, but we can't obviously factor one out of the bottom. We need to use an algebraic trick make the $x-1$ appear. What tricks do we have that might work?

(c) What is $\lim_{X\to 1} h(x)$?

Problem 6. Let

$$f(x) = \begin{cases} x^2+1 & x > 2 \\ 9-2x & x < 2 \end{cases}$$

Can we extend this to a continuous function on all reals?

(a) Where is f continuous? Where is it discontinuous?

(b) What value "should" $f(x)$ have for $x = 2$?

(c) Can you define a function that's Almost Identical to $f(x)$, but continuous at all reals?